Review of Physics 2 - Exam, muster

Semestr: *sommer, 20XX/XX,* ***Tutor:*** *Martin Žáček,* ***Date:*** *20XX-XX-XX*

For every task is for correct general result 1 point, for correct numerical result 1 point and correct way of solution for 3 points, i.e. maximum of possible points is 5 per task and maximum 20 points for the test. Numerical results estimate with the 1-digit of precision.

Task 1 - Water power plant

Calculate the difference in the river levels before and after the hydroelectric power station, where the current flow rate is $I = 400\ \mathrm{m^3 s^{-1}}$, the output power is $P = 2\ \mathrm{MW}$, and efficiency of the power plant is $\eta = 75\ \%$. Assume gravitational acceleration as $g = 10\ \mathrm{m\,s^{-2}}$.

Solution:

$$\Delta W = mg\Delta h;$$

$$P_{\mathrm{water}} = \frac{\Delta W}{t} = \frac{mg\Delta h}{t} = \rho I g \Delta h;$$

$$P_{\mathrm{out}} = \eta P_{\mathrm{water}};$$

$$\Delta h = \frac{P_{\mathrm{out}}}{I\rho g\eta} = \frac{2\cdot 10^6}{400\cdot 10\cdot 1\,000\cdot 0.75} = \frac{2}{3}\ \mathrm{m}.$$

Task 2 - Linear harmonic oscillator

The linear harmonic oscillator with the amplitude of displacement $y_0 = 12\ \mathrm{cm}$ has a period of movement $T = 40\ \mathrm{ms}$. The rigidity of the oscillating system is $k = 6\ \mathrm{Nm^{-1}}$. Calculate the total mass, maximal velocity, maximal acceleration, and total energy of the oscillator.

Solution:

$$m = \frac{k}{\omega^2} = \frac{T^2 k}{4\pi^2} = \frac{0.04\cdot 0.04\cdot 6}{40} = 24\cdot 10^{-5}\ \mathrm{kg} = 210\ \mathrm{mg};$$

$$v_{\mathrm{max}} = \omega y_0 = \frac{2\pi}{T} y_0 \approx \frac{2\cdot 3.14}{0.04}\cdot 0.12 = 2\cdot 3.14\cdot 3 \approx 19\ \mathrm{m\,s^{-1}};$$

$$a_{\mathrm{max}} = \omega v_{\mathrm{max}} = \omega^2 y_0 = \frac{4\pi^2}{T^2} y_0 \approx \frac{2\cdot 3.14}{0.04}\cdot 19 \approx 3\,000\ \mathrm{m\,s^{-2}};$$

$$E_{\mathrm{tot}} = E_{\mathrm{k\,max}} = \frac{1}{2}\frac{k}{\omega^2}(\omega y_0)^2 = \frac{1}{2} k y_0^2 = \frac{1}{2}\cdot 6\cdot 0.12^2 \approx 3\cdot 0.014\,4 \approx 0.04\ \mathrm{J}.$$

Task 3 - Capacitors

Three capacitors with capacities $C_1 = 1\ \mu\text{F}$, $C_2 = 3\ \mu\text{F}$ and $C_3 = 20\ \mu\text{F}$ are connected serial. What is the total capacity? What is the total bound electric charge, if the capacities are charged to voltage $U = 200\ \text{V}$? how is the voltage divided into individual capacities?

Solution:

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{\mu F}\left(\frac{1}{1} + \frac{1}{3} + \frac{1}{20}\right)} = \frac{1 \mu F}{\frac{60+20+3}{60}} = \frac{60}{83}\ \mu F;$$

$$Q = CU = \left(\frac{60}{83}\ \mu F\right) \cdot 200\ \text{V} \approx 145\ \mu C;$$

$$U_1 = \frac{Q}{C_1} = \frac{UC}{C_1} = U\frac{\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}}{C_1} = U\frac{1}{\frac{C_1}{C_1} + \frac{C_1}{C_2} + \frac{C_1}{C_3}} = 200\frac{1}{1 + \frac{1}{3} + \frac{1}{20}} = 200\frac{60}{83} \approx 145\ \text{V};$$

$$U_2 = U\frac{1}{\frac{C_2}{C_1} + \frac{C_2}{C_2} + \frac{C_2}{C_3}} = 200\frac{1}{3 + 1 + \frac{3}{20}} = 200\frac{20}{83} \approx 48\ \text{V};$$

$$U_3 = U\frac{1}{\frac{C_3}{C_1} + \frac{C_3}{C_2} + \frac{C_3}{C_3}} = 200\frac{1}{20 + \frac{20}{3} + 1} = 200\frac{3}{81} \approx 7\ \text{V};$$

Task 4 - Water vapor

Inside the closed glass tube with a volume of $V = 3\ \text{cm}^3$ is located pure water vapor with the pressure $p = 2\,000\ \text{Pa}$ and the temperature $\vartheta = 20\ ^\circ\text{C}$. Calculate their total mass, molar mass, density and number of molecules of the vapor. Relative atomic mass let's assume as 1 for hydrogen and 16 for oxygen; the molar gas constant is $R = 8.3\ \text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$, Avogadro constant is $N_\text{A} = 6.6 \cdot 10^{23}\ \text{mol}^{-1}$, let's assume the behavior of the vapor as an ideal gas.

$$M = \frac{m}{n} = \frac{M m_\text{molecule}}{\frac{N}{N_\text{A}}} = N_\text{A} u \left(2A_\text{H} + A_\text{O}\right) = \frac{1\ \text{g}}{1\ \text{mol}}\left(2 \cdot 1 + 16\right) = 18\ \text{g}\,\text{mol}^{-1};$$

$$pV = nRT = \frac{m}{M}RT; T = \left(\vartheta + 273.15\right)\ \text{K} \approx 293\ \text{K};$$

$$m = \frac{pVM}{RT} \approx \frac{2\,000 \cdot 3 \cdot 10^{-6} \cdot 18\ \text{g}\,\text{mol}^{-1}}{8.3 \cdot 293} \approx \frac{9}{2} \cdot 10^{-5}\ \text{g} \approx 45\ \text{ng};$$

$$\rho = \frac{m}{V} = \frac{pM}{RT} \approx \frac{2\,000 \cdot 18\ \text{g}\,\text{mol}^{-1}}{8.3 \cdot 293} \approx \frac{30}{2}\ \text{g}\,\text{m}^{-3} = 15\ \text{g}\,\text{m}^{-3};$$

$$N = nN_\text{A} = \frac{pVM}{RT}N_\text{A} \approx \frac{2\,000 \cdot 3 \cdot 10^{-6}}{8.3 \cdot 293} \cdot 6 \cdot 10^{23} \approx 1.5 \cdot 10^{18}.$$