# Review of Physics 2 - Exam

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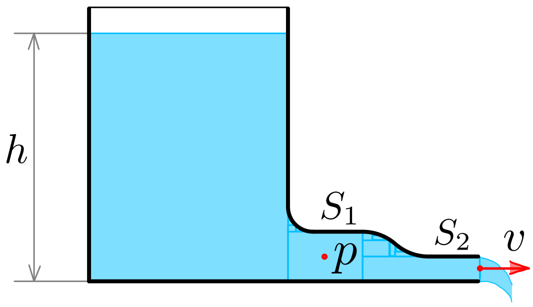
## Task 1 - Water flowing out of a vessel

A vessel is filled with water to the height $h = 0.5\ \mathrm{m}$. At the bottom is connected a horizontal tube consisting of two parts. The first one has cross-section $S_1 = 1\ \mathrm{cm}^2$, the second one $S_2 = 0.5\ \mathrm{cm}^2$ and its end is open so water can flow freely out of the vessel (see picture).




1. Calculate the speed $v$ of the outflowing water at the end of the tube.
2. Calculate the pressure $p$ in the first part of the tube.

Calculate with the density of water $\rho = 1\,000\ \mathrm{kg\,m^{-3}}$ and the gravity acceleration $g = 10\ \mathrm{m\,s^{-2}}$.

**Solution:**

Let us denote the required quantities $v$ and $p$.

1. From Bernulli equation we get from comparison of points at the level and at the end of the tube

$$\rho g h = \frac{1}{2}\rho v^2 \rightarrow v = \sqrt{2gh};$$

$$v = \sqrt{2 \cdot 10\ \mathrm{m\,s^{-2}} \cdot 0.5\ \mathrm{m}} = \sqrt{10}\ \mathrm{m\,s^{-1}} \approx 3\ \mathrm{m\,s^{-1}}.$$

2. Let's mark the velocity in the first part of the tube as $v_1$. From Bernulli equation we get

$$\rho g h = p + \frac{1}{2}\rho^2 v_1^2;$$

relationship between velocities $v_1$ and $v$ in the first and second part of the horisontal tube is given by continuity equation $v_1 S_1 = v S_2$, so we have

$$v_1 = v\left(\frac{S_2}{S_1}\right),$$

let's substitute into the first equation and express and use the equation from the point 1. and we get:

$$p = \rho g h \left(1 - \frac{S_2}{S_1}\right)$$

and numericaly we get

$$p = 1\,000\ \mathrm{kg\,m^{-3}} \cdot 10\ \mathrm{m\,s^{-2}} \cdot 0.5\ \mathrm{m} \left(1 - \frac{0.5}{1}\right) = 2.5\ \mathrm{kPa}.$$

## Task 2 - The seconds pendulum

The seconds pendulum is a pendulum whose length is set so that the period is equal to 2 seconds.

1. Calculate the length of the seconds pendulum, assume gravitational acceleration as $g = 10\ \mathrm{m\,s^{-2}}$.
2. Calculate the total energy of the second pendulum, if the mass is equal $m = 2$ kg and the initial deflection is $\varphi_0 = 2^\circ$
3. What is the ratio of the lengths $l_\mathrm{E}/l_\mathrm{M}$ and total energies $E_\mathrm{E}/E_\mathrm{M}$ for pendulums located on Earth and the Moon with equal initial deflections $\varphi_0 = 2^\circ$? Assume the ratio of accelerations as $g_\mathrm{E}/g_\mathrm{M} = 9.81/1.62 = 6.056 \approx 6$.

**Solution:**

1. $$\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{g}{l} \rightarrow l = \frac{gT^2}{4\pi^2} \approx \frac{10 \cdot 2^2}{4 \cdot 10} = 1\ \mathrm{m}.$$

2. $$E_\mathrm{tot} = \frac{1}{2} m \omega^2 x_0^2 = \frac{1}{2} m \frac{g}{l} (l\alpha_0)^2 = \frac{1}{2} m g l \alpha_0^2 = \frac{m g^2 T^2 \alpha_0^2}{8\pi^2}$$

   numerical value can be more effectively calculateble from

$$E_\mathrm{tot} = \frac{1}{2} m g l \alpha_0^2 = \frac{2\ \mathrm{kg} \cdot 10\ \mathrm{m\,s^{-2}} \cdot 1\ \mathrm{m}}{2} \left(2 \frac{2\pi}{360}\right)^2;$$

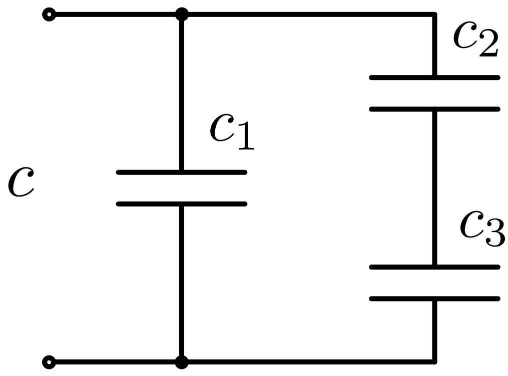
   and the final numerical result is

$$E_\mathrm{tot} = 2 \left(\frac{3.14}{90}\right)^2 \mathrm{J} \approx 0.0122\ \mathrm{J} \approx 10\ \mathrm{mJ}.$$

3. $$\frac{l_\mathrm{E}}{l_\mathrm{M}} = \frac{g_\mathrm{E} \cancel{T^2}\ \cancel{4\pi^2}}{g_\mathrm{M} \cancel{T^2}\ \cancel{4\pi^2}} = 6;$$

$$\frac{E_\mathrm{E}}{E_\mathrm{M}} = \frac{\cancel{2}\ \cancel{m}\ g_\mathrm{E} l_\mathrm{E}\ \cancel{\alpha_0^2}}{\cancel{2}\ \cancel{m}\ g_\mathrm{M} l_\mathrm{M}\ \cancel{\alpha_0^2}} = 36.$$

## Task 3 - Capacitors




Three capacitors with capacities $C_1 = 5\ \mu\text{F}$, $C_2 = 3\ \mu\text{F}$ and $C_3 = 2\ \mu\text{F}$ are connected serio-paralell where the first one is connected paralel with the remaining two, which are connected in series (see picture). Initially, the capacitors were not charged. Then was connected to a source with voltage $U = 10$ V.

1. Calculate the total capacity.
2. Calculate the voltage at the capacitor $C_3$.
3. Calculate the total bound charge of all capacitors.

**Solution:**

1.
$$C = C_1 + \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = C_1 + \frac{C_2 C_3}{C_1 + C_3};$$

$$C = \left(5 + \frac{3 \cdot 2}{3 + 2}\right)\ \mu\text{F} = \frac{31}{5}\ \mu\text{F} \approx 6\ \mu\text{F}.$$

2. Capacities and charges satisfy the following set of equations (from definition of the capacity)

$$Q_1 = C_1 U_1; Q_2 = C_2 U_2; Q_3 = C_3 U_3$$

and because the capacities $C_2$ and $C_3$ are connected serial and the charge that flowed through both is the same,

$$Q_2 = Q_3 \to C_2 U_2 = C_3 U_3;$$

$$U = U_1 = U_2 + U_3;$$

and from last two equations we get

$$U_3 = U \frac{C_2}{C_2 + C_3} = 10\ \text{V} \cdot \frac{3}{3 + 2} = 6\ \text{V}.$$

3.
$$Q = Q_1 + Q_2 + Q_3 = Q_1 + 2Q_3 = U\left(C_1 + 2C_3 \frac{C_2}{C_2 + C_3}\right);$$

$$Q = 10\ \text{V}\left(5 + 2 \cdot 3 \frac{2}{3 + 2}\right)\ \mu\text{F} = 74\ \mu\text{C}.$$

## Task 4 - Weight of the atmosphere

From the pressure acting to the Earth's surface calculate

1. The total weight $m$ of the atmosphere.
2. The total amount of matter $s$ of the atmosphere.
3. The total amount of particles $N$ in the the atmosphere.
4. The teoretical height $h$ of the the atmosphere with assumption that their concentration $n = \frac{N}{V} = \text{const.}$

Assume the behavior of the atmosphere as an ideal gas with the constant pressure $p = 10^5$ Pa and the constant temperature $\vartheta = 20\ {}^\circ\mathrm{C}$. Calculate with the Earth's radius $R = 64 \cdot 10^5$ m. Relative atomic mass use as 14 and 16 for nitrogen and oxygen respectively; atmosphere take as compoud of two-atomic molecules with the N:O ratio as 4:1; the molar gas constant is $R_\mathrm{m} = 8.3\ \mathrm{J\,K^{-1}\,mol^{-1}}$ and Avogadro constant is $N_\mathrm{A} = 6.6 \cdot 10^{23}\ \mathrm{mol^{-1}}$.

**Solution:**

1.
$$F = mg \rightarrow m = \frac{F}{g} = \frac{pS}{g} = \frac{p \cdot 4\pi R^2}{g}$$

$$m = \frac{10^5\ \mathrm{Pa}}{10\ \mathrm{m\,s^{-2}}} \cdot 4 \cdot 3.14 \cdot 64^2 \cdot 10^{10}\ \mathrm{m^2} \approx 5 \cdot 10^{18}\ \mathrm{kg}.$$

2. Molar mass of the ear is the weighted mean value of molar masses of individual components,

$$M = \frac{4M_{\mathrm{N\_2}} + 1 \cdot M_{\mathrm{O\_2}}}{4+1} = \frac{4 \cdot 28 + 32}{5} = 28.8\ \mathrm{g\,mol^{-1}}$$

$$M = \frac{m}{s} \rightarrow s = \frac{m}{M} \approx \frac{5 \cdot 10^{18}\ \mathrm{kg}}{28.8\ \mathrm{g\,mol^{-1}}} \approx 179 \cdot 10^{20}\ \mathrm{mol} \doteq 2 \cdot 10^{22}\ \mathrm{mol}.$$

3. $N = sM_A = 179 \cdot 10^{20}\mathrm{mol} \cdot 6.6 \cdot 10^{23}\ \mathrm{mol^{-1}} \approx 1.18 \cdot 10^{46} \approx 1 \cdot 10^{46}$ molecules.

4.
$$V = Sh \rightarrow h = \frac{V}{S} \underset{pV = sR_\mathrm{m}T}{\overset{\uparrow}{=}} \frac{sR_\mathrm{m}T}{pS} = \frac{\cancel{p} \cdot \cancel{4\pi R^2}\, R_\mathrm{m}T}{\cancel{p}\, g \cdot \cancel{4\pi R^2}\, M} = \frac{R_\mathrm{m}T}{gM};$$

$$h = \frac{8.3\ \mathrm{J\,K^{-1}\,mol^{-1}} \cdot 293\ \mathrm{K}}{10\ \mathrm{m\,s^{-2}} \cdot 28.8\ \mathrm{g\,mol^{-1}}} \approx 8\ \mathrm{km}.$$