

1. Hydromechanics

Notes to the theory, Review of physics 2

1.1. Hydrostatics

Pressure:

$$P = \frac{F}{A} \quad \begin{array}{l} P \text{ pressure on surface,} \\ F \text{ the force acting at right angle to the surface} \\ A \text{ the area over which the force is acting} \end{array} \quad \begin{array}{l} (\text{N/m} = \text{Pa}) \\ (\text{N}) \\ (\text{m}^2) \end{array}$$

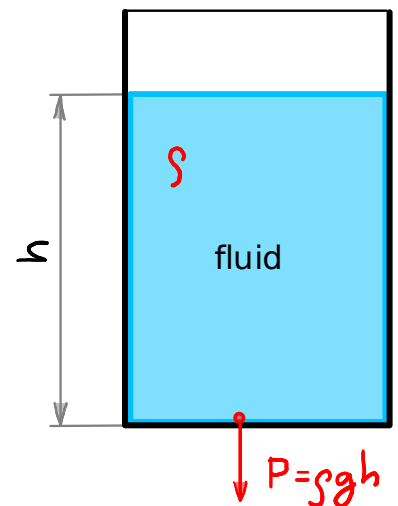
Pa: the pascal

Density:

$$\rho = \frac{m}{V} \quad \begin{array}{l} \rho \text{ density of substance } (\text{kg/m}^3) \\ m \text{ mass of substance } (\text{kg}) \\ V \text{ volume of substance } (\text{m}^3) \end{array}$$

Hydrostatic pressure:

$$p = \rho g h \quad \begin{array}{l} g \text{ gravity acceleration } (\approx 9.81 \text{ m/s}^2) \\ h \text{ deep } (\text{m}) \end{array}$$

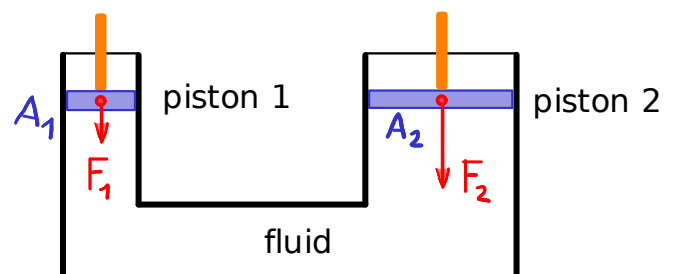


Pascal's Principle:

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

Press machine:

$$\begin{aligned} P_1 &= P_2 \quad (\text{Pascal's Principle}) \\ \frac{F_1}{A_1} &= \frac{F_2}{A_2} \\ F_2 &= \frac{A_2}{A_1} F_1 \end{aligned}$$

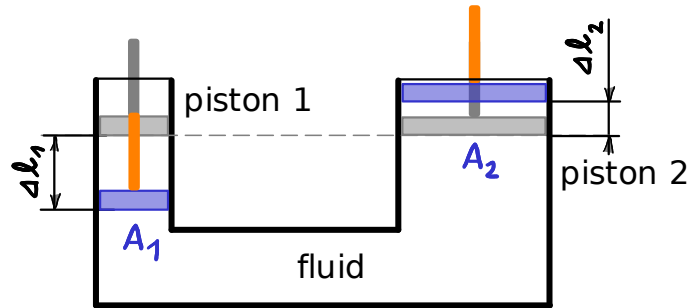


$$V_1 = V_2 \quad (\text{incompressibility})$$

$$A_1 \Delta l_1 = A_2 \Delta l_2$$

$$\Delta l_2 = \frac{A_1}{A_2} \Delta l_1$$

Δl ... displacement



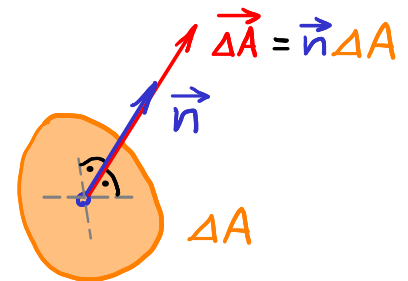
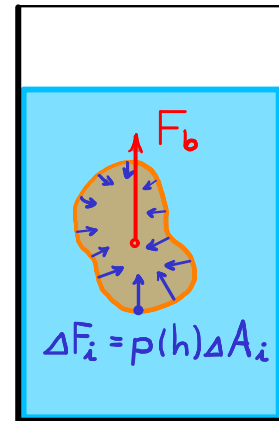
Buoyant force:

... is the vector sum of external forces acting due to hydrostatic pressure.

$$\vec{F}_b = \sum_i \Delta \vec{F}_i = \sum_i (-p(h) \vec{\Delta A}_i)$$

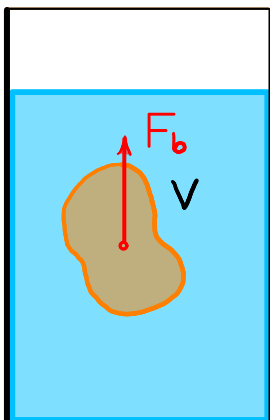
where sum is over the whole surface.

The area as a vector is defined as area in magnitude multiplied by normal vector. Normal vector is oriented by convention outward the volume where the pressure acts inwards, so the negative sign must be in the summation.

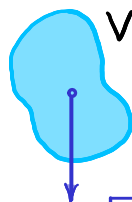


Archimedes Principle:

The buoyant force on an object immersed in a fluid (or floating in a liquid) is equal in magnitude to the weight of the fluid "displaced" by the object.



$$F_b = F_e$$



$$F_e = mg = \rho_f V g$$

ρ_f ... density of the fluid

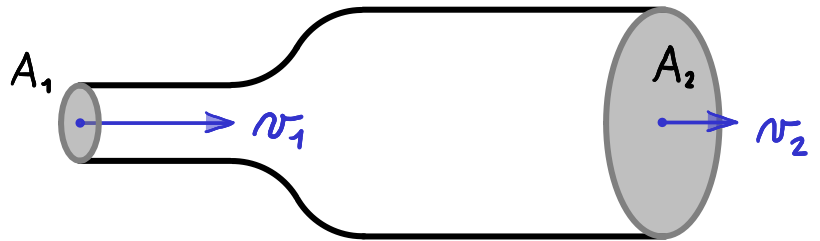
1.1. Hydrodynamics

Continuity equation:

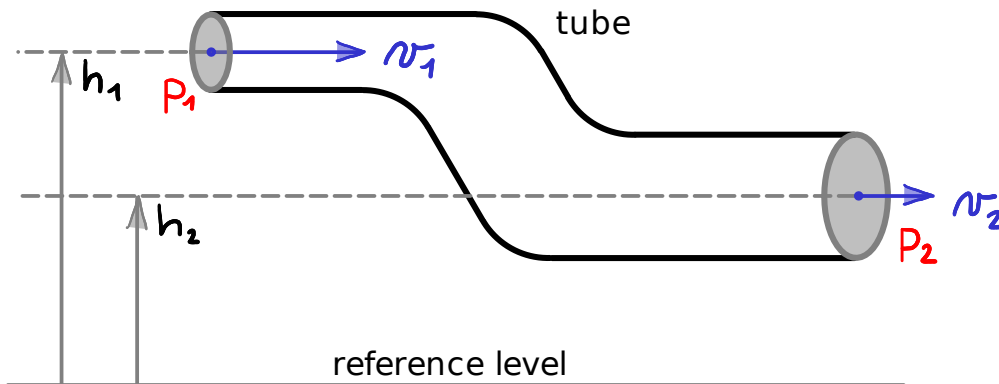
$$V_1 = V_2 \text{ (incompressibility)}$$

$$A_1 v_1 \cancel{dt} = A_2 v_2 \cancel{dt}$$

$$A_1 v_1 = A_2 v_2$$



Bernulli equation:



In a tube with flowing liquid the pressure, velocity and level change at each point but the following their combination remains constant:

$$\frac{1}{2} \rho v^2 + \rho g h + p = \text{const.}$$

In fact this is the energy conservation law expressed in their densities.

Following assumptions must be fullfield:

- The fluid is incompressible.
- The friction against the walls of the tube is negligible.
- The friction of the fluid itself (the viscosity) is negligible.
- The flow is laminar.

If we compare two points of the tube, we get the Bernulli equation:

$$\frac{1}{2} \rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2 + P_2$$