Kmity

Příklad 10.2

Harmonický pohyb, $v_{max} = 6\ m/s$, $a_{max} = 24\ m/s^2$

a) $\omega = ?$ b) $T = ?$ c) $f = ?$ d) $X_0 = ?$

Řešení:

$$X(t) = X_0 \cos(\omega t + \varphi) =$$
$$= A\cos(\omega t) + B\sin(\omega t) =$$
$$= \mathrm{Re}\{X_0 e^{j(\omega t+\varphi)}\} \underset{\text{konvence}}{=}$$
$$= X_0 e^{j(\omega t+\varphi)} =$$
$$= \underbrace{X_0 e^{j\varphi}}_{\hat{X}} e^{j\omega t} = \hat{X} e^{j\omega t}$$

$$v(t) = \dot{X} = -X_0 \omega \sin(\omega t + \varphi)$$
$$v_{max} = X_0 \omega$$
$$a(t) = \dot{v} = -X_0 \omega^2 \cos(\omega t + \varphi)$$
$$a_{max} = X_0 \omega^2$$

zadáno:

2 rovnice pro X_0, ω
$$\begin{cases} X_0 \omega = v_{max} & (1) \\ X_0 \omega^2 = a_{max} & (2) \end{cases}$$

$$(1)\quad X_0 = \frac{v_{max}}{\omega} \xrightarrow[\text{do (2)}]{} \frac{v_{max}}{\cancel{\omega}} \omega^{\cancel{2}} = a_{max}$$

$$\omega = \frac{a_{max}}{v_{max}}, \quad X_0 = \frac{v_{max}}{\omega} = \frac{v_{max}^2}{a_{max}}$$

a) $$\omega = \frac{24\ m/s^2}{6\ m/s} = 4\ s^{-1}$$

d) $$X_0 = \frac{6\ m/s}{4\ s^{-1}} = 1.5\ m$$

b) $$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\ s^{-1}} = \frac{\pi}{2}\ s$$

c) $$f = \frac{\omega}{2\pi} = \frac{2}{\pi}\ s^{-1}$$

Příklad 10.3

Těleso na pružině, $T = 0.5\,s$, $g = 9{,}81\,m/s^2$.
Okolik se pružina zkrátí odstraněním tělesa?

Řešení

$F_r = -kx$ $\quad$ k ... tuhost $\quad$ $\varepsilon = \frac{l - l_0}{l_0} = \frac{\Delta l}{l}$

$\omega^2 = \frac{k}{m}$ $\quad$ $\ddot{x} + \omega^2 x = 0$ $\quad$ $\sigma = E\varepsilon$ Hookův zákon

$k = \omega^2 m$ $\quad$ $\sigma = \frac{F}{S}$

$F_r = mg$ ... závaží volně visí

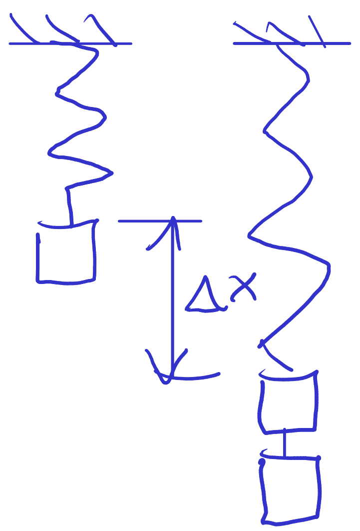
$= kx_1 = \omega^2 m x_1$

Rovnice $\quad \cancel{m}g = \omega^2 \cancel{m} x_1$

$$x_1 = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2} \approx \frac{10\,\frac{m}{s^2}\left(\frac{1}{2}\,s\right)^2}{4\pi^2} \approx \frac{\cancel{10}}{16 \cdot \cancel{10}} \approx 0.06\,m$$

$\sqrt{10} \approx 3.16 \rightarrow \pi^2 \approx 10$

V laboratoři



$$k\Delta x = g\,\Delta m$$

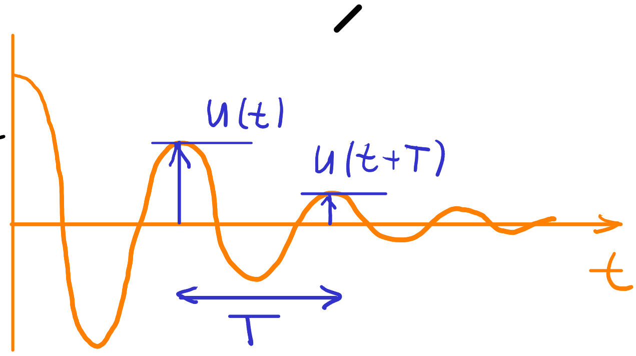
$$k = g\,\frac{\Delta m}{\Delta x}$$

Příklad 10.6

Za jak dlouho se energie kmitavého pohybu ladičky s frekvencí $f = 435\,\mathrm{Hz}$ zmenší 10^6x? $Q = ?$

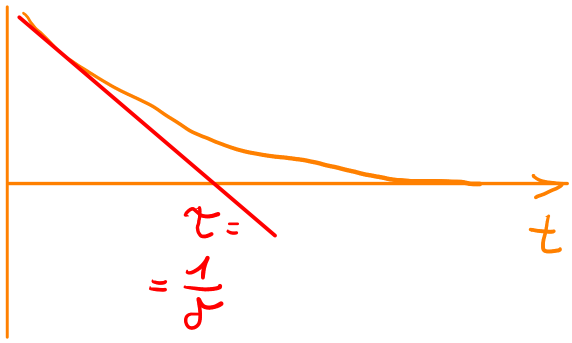
$\Lambda = 8\cdot 10^{-4}$

$$\Lambda = \ln \frac{u(t)}{u(t+T)} = \ln \frac{\cancel{u_0} e^{-\delta t} \cancel{\cos(\omega t+\varphi)}}{\cancel{u_0} \underbrace{e^{-\delta(t+T)}}_{e^{-\delta T} e^{-\delta t}} \cos(\omega t+\varphi)} = \ln e^{\delta T}$$



$$u(t) = u_0 e^{-\delta t}$$

$$\Lambda = \delta T = \frac{T}{\tau} = \frac{\delta}{f}$$



$$\delta = f\Lambda$$

$$E \propto u^2 \qquad \left(E_k = \frac{1}{2} m v^2 \quad E_p = \frac{1}{2} k u^2 \right)$$

$$n = \frac{E(t)}{E(t+t_1)} = \frac{u^2(t)}{u^2(t+t_1)} = \frac{\cancel{u_0^2} \cancel{e^{-2\delta t}}}{\cancel{u_0^2} e^{-2\delta(\cancel{t}+t_1)}} =$$

$$= e^{2\delta t_1} \quad \rightarrow \quad \ln n = 2\delta t_1$$

$$t_1 = \frac{\ln n}{2\delta} = \frac{\ln n}{f\Lambda} =$$

$$\ln n = \frac{\log n}{\log e}$$

$$= \frac{\log 10^6}{435\,\mathrm{s}^{-1} \cdot 8\cdot 10^{-4} \log e} =$$

$$Q \overset{def}{=} 2\pi \frac{W(t)}{W(t) - W(t+T)}$$

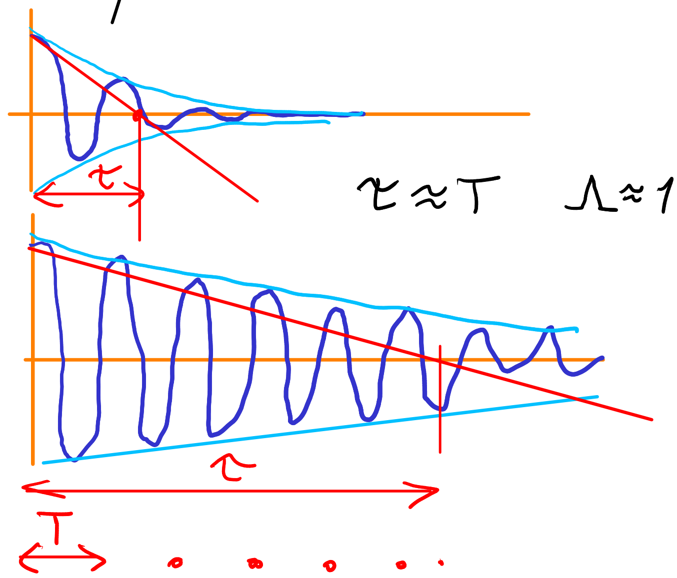
← Energie

← Disipovaná energie

$$= 2\pi \frac{u_0^2 e^{-2\delta t}}{u_0^2 e^{-2\delta t} e^{-2\delta T}} = 2\pi \cdot e^{2\delta T} =$$

$$= 2\pi e^{2\cancel{f}\Lambda\cancel{T}} = \underline{\underline{2\pi e^{2\Lambda}}}$$

význam Λ:



$\tau \approx T \quad \Lambda \approx 1$

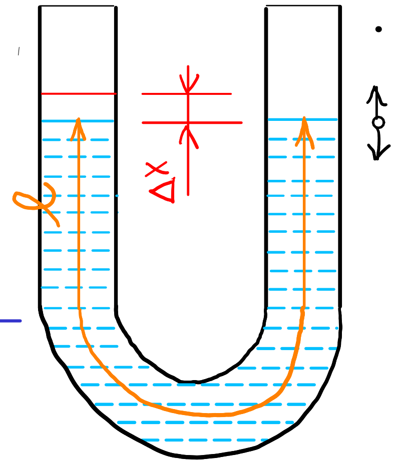
$\tau \approx 5.5T \quad \Lambda \approx \frac{1}{5.5} \approx 0.18$

LHO $\Lambda = 0$

Příklad 10.10

Kmitavý pohyb kapaliny v U trubici.

$T = ?$



$$\omega^2 = \frac{k}{m} \qquad k = ?$$

$$F = 2\Delta x \cdot \rho S g = 2\rho \underbrace{\underbrace{S\Delta x}_{V}}\, g = \underbrace{2\rho S g}_{k} \cdot \Delta x$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2\cancel{\rho}\cancel{S}g}{\cancel{\rho}\cancel{S}\ell}} = \boxed{\sqrt{\frac{2g}{\ell}}}$$

$m = \rho S \ell$

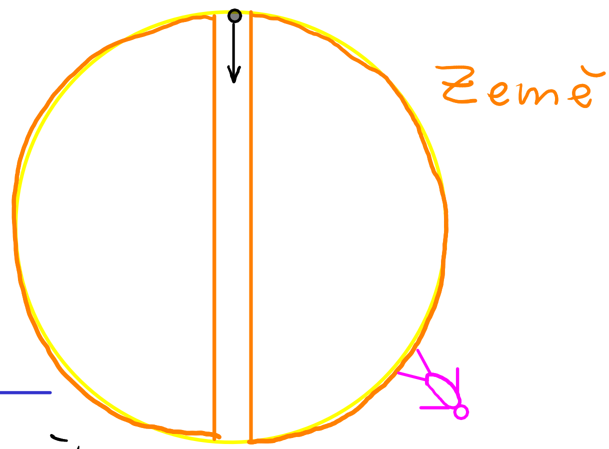
rozměrová kontrola

$$\left[\sqrt{\frac{2g}{\ell}}\right] = \left(\frac{\cancel{m}}{s^2\,\cancel{m}}\right)^{\frac{1}{2}} = \frac{1}{s}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{2g}} = \pi\sqrt{\frac{2\ell}{g}} \approx \pi\sqrt{\frac{2\cdot 1\,m}{10\,m\,s^{-2}}} \approx 1{,}4\,s$$

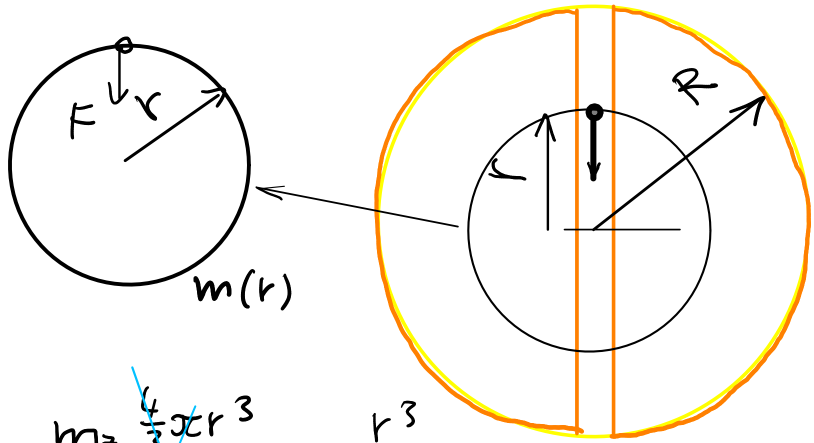
Příklad 10.11

Za jak dlouho se těleso spadlé do tunelu v Zemi vrátí? Země nechť je homogenní.

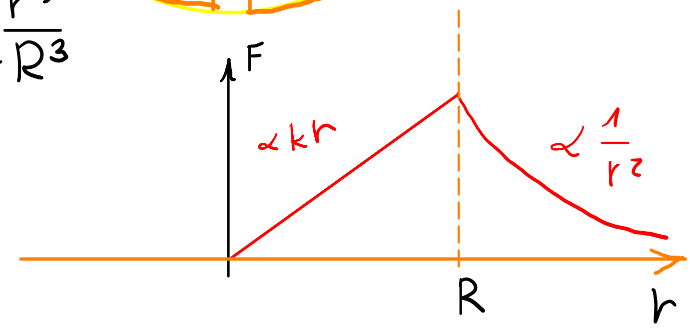


Lze ukázat (viz Gaussova věta elektrostatiky)

$$F = G\frac{m\, m(r)}{r^2} =$$



$$m(r) = \rho \frac{4}{3}\pi r^3 = \frac{m_Z \cancel{\frac{4}{3}\pi} r^3}{\cancel{\frac{4}{3}\pi} R^3} = m_Z \frac{r^3}{R^3}$$



$$F = G\frac{m\, m_Z\, r^{\cancel{3}}}{\cancel{r^2}\, R^3} = \underbrace{G\frac{m\, m_Z}{R^3}}_{k}\, r$$

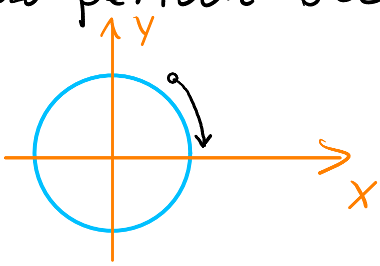
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\cancel{m} R^3}{G \cancel{m} m_Z}} = 2\pi\sqrt{\underbrace{\frac{R^2}{G m_Z}}_{1/g} R} =$$

$$\cancel{m} g = \frac{G \cancel{m} m_Z}{R^2}$$

$$\approx 2\pi\sqrt{\frac{1}{10\,\mathrm{m/s^2}}\, 6\,400\cdot 10^3\,\mathrm{m}} = 2\pi \underbrace{\sqrt{64}}_{8}\sqrt{10^4} = 2\pi\cdot 800 \doteq$$

$$\doteq 4800\,\mathrm{s} = 80\,\mathrm{min}$$

Rovno periodě oběhu družice na nízké oběžné dráze.



$$F_x = m a_x \qquad F_y = m a_y$$

$$x = R\cos\omega t \qquad y = R\sin\omega t$$

Příklad 10.15

$u = u_1 + u_2$ $\quad u_1 = A_1 \sin(\omega t + \varphi_1)$, $u_2 = A_2 \sin(\omega t + \varphi_2)$

$A_1 = 3\,cm$, $A_2 = 5\,cm$, $\varphi_1 = 0°$, $\varphi_2 = 60°$.

$A = ?$, $\varphi = ?$

Příklad 10.17

Nalezněte rovnici dráhy tělesa, jehož pohyb vznikl složením vzájemně kolmých kmitů,
$A_1 = 10\,cm$, $A_2 = 5\,cm$, $\varphi_1 = \varphi_2 = \varphi_0$.

Příklad 10.18

Příklad 10.19

Hmotný bod, $x = A\cos\omega t$
$y = B\sin\omega t$

a) Tvar trajektorie ?

b) $\vec{v}$ c) v

d) $\vec{a}$ e) a