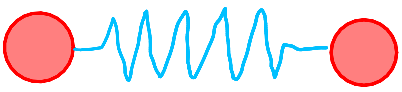
Dnes: kmity, pokračování

LHO shrnutí: $F_r = -kx \qquad \ddot{x} + \omega^2 x = 0$

$$\omega^2 = \frac{k}{m} \qquad \left(\frac{1}{LC} \dots\right)$$

Příklad 9.3 $\qquad W_p = W_0\left(1 - e^{-\alpha(r-r_0)^2}\right)$

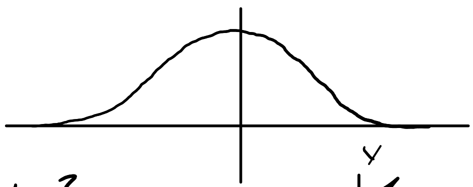


a) Nakreslete W_p

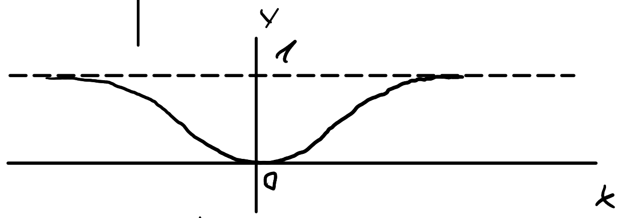
b) $\omega^* = ?$

↑ frekvence malých kmitů

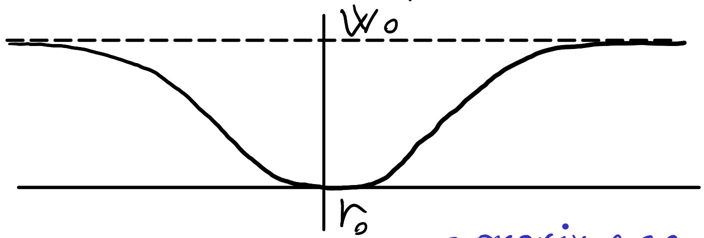
Řešení: a) $e^{-\alpha x^2}$



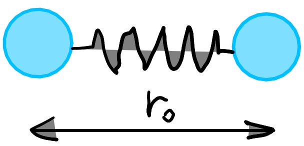
$1 - e^{-\alpha x^2}$

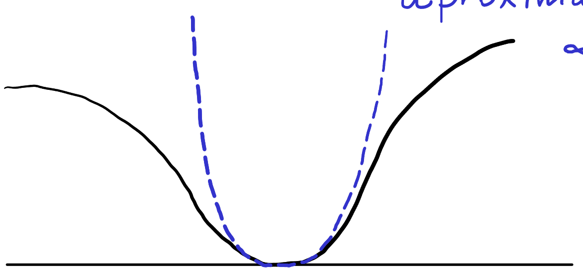


$W_p(r)$



b) r_0 je minimum





Taylorův rozvoj: $\displaystyle f(x-x_0) = \sum_{k=0}^{\infty} \frac{1}{k!}\frac{d^k f}{dx^k}(x-x_0)^k =$

$$= f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \dots$$

zde f je $\quad W_p = W_0\left(1 - e^{-\alpha(r-r_0)^2}\right)$

$$W_p'(r) = -W_0 e^{-\alpha(r-r_0)^2} \cdot (-2\alpha(r-r_0))\underbrace{r'}_{1} = 2\alpha W_0 (r-r_0) e^{-\alpha(r-r_0)^2}$$

$W_p(r_0) = 0$ (to jsme očekávali; jde o minimum)

$$W''(r) = 2\alpha W_0 \left[e^{-\alpha(r-r_0)^2} + (r-r_0) e^{-\alpha(r-r_0)^2} (-2\alpha(r-r_0)) \right] =$$
$$= 2\alpha W_0 e^{-\alpha(r-r_0)^2} (1 - (r-r_0)^2 \cdot 2\alpha)$$

$$W''(r_0) = 2\alpha W_0$$

Výsledný potenciál v aproximaci Taylorova rozvoje do kv. členu:

$$W_p = \frac{1}{2} W''(r_0)(r-r_0)^2 = \frac{1}{\cancel{2}} \underbrace{\cancel{2}\alpha W_0}_{W_p''(r_0)} (r-r_0)^2$$

Musíme najít tuhost.

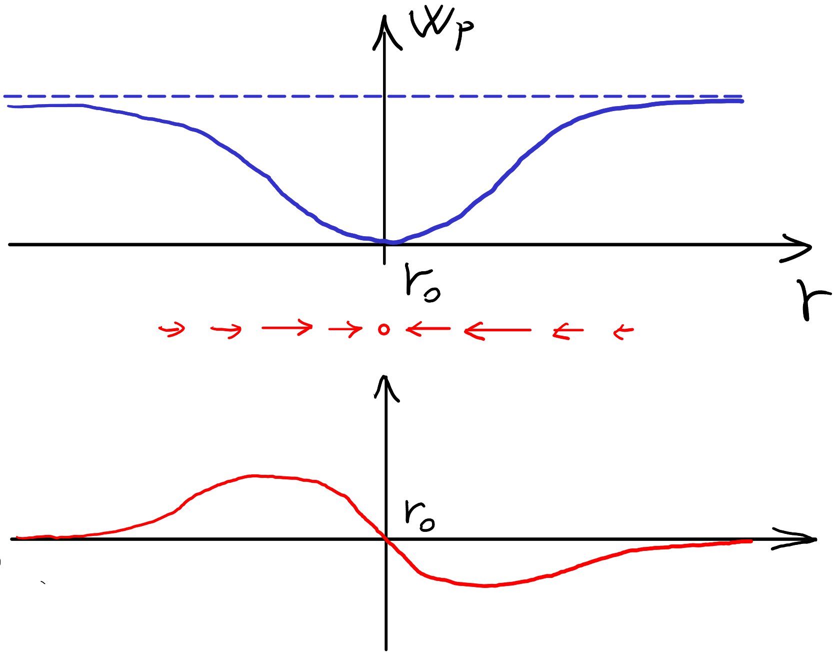
$$F_r = -k\underbrace{(r-r_0)}_{x} \rightarrow W_p = -\int F_r\, dr = k\frac{(r-r_0)^2}{2}$$

$$\int x\, dx = \frac{x^2}{2} + K$$

Porovnáme: $k = 2\alpha W_0$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2\alpha W_0}{m}}$$

síly:



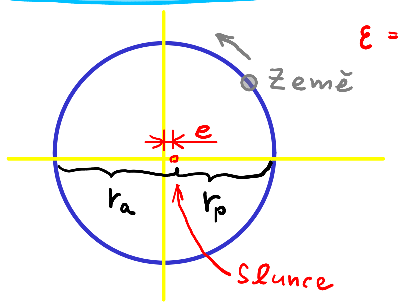
$$F = -\frac{dW_p}{dr}$$

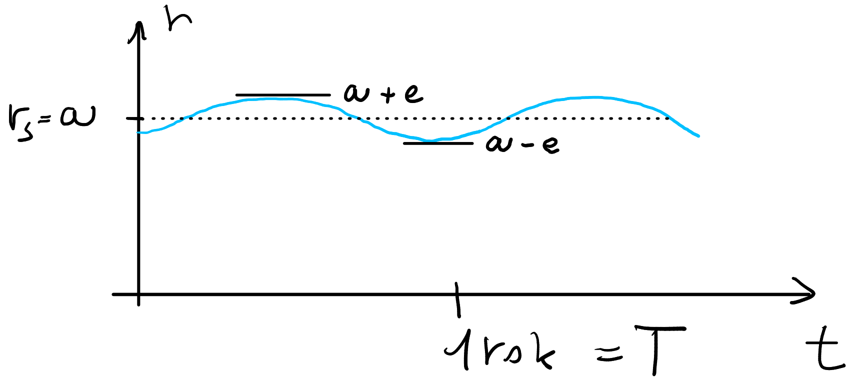
Síla míří vždy k lokálnímu minimu energie.

Všimněte si také, že v blízkosti minima je $F \approx -k(r-r_0)$.

Příklad 9.4 Země jako LHO

$\varepsilon = \frac{e}{a} = \frac{1}{60}$





$$E = \underbrace{E_k}_{\frac{1}{2}mv^2} + E_p = \frac{1}{2}m\dot{r}^2 + \underbrace{\frac{1}{2}mr^2\omega^2 - G\frac{m_s m}{r}}$$

$v^2 = v_x^2 + v_y^2 = v_r^2 + v_\varphi^2$

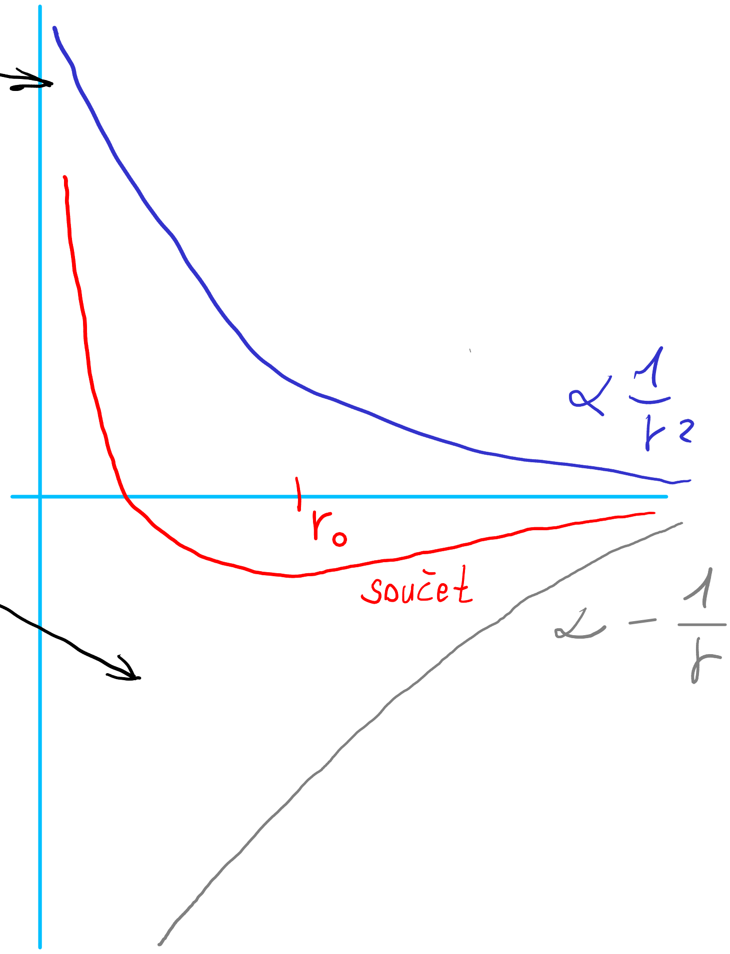
závisí na r, formálně můžeme považovat za potenciál.

$$W_{rot} = \frac{1}{2}J\omega^2 = \frac{1}{2}mr^2\omega^2$$

$$b = r v_\varphi = r^2 \omega m = konst. \rightarrow \omega = \frac{b}{r^2 m}$$

$$W_{eff} = \frac{1}{2}m^{-1}\cancel{r^2}\frac{b^2}{\cancel{r^4}\,2} - G\frac{m_s m}{r}$$

$$= \frac{A}{r^2} - \frac{B}{r}$$



$$W'_{eff} = -2Ar^{-3} + Br^{-2}$$

Minimum:

$$2A\cancel{r^{-3}}^{\,-1} = B\cancel{r^{-2}}$$

$$r_0 = \frac{2A}{B}$$

$$W''_{eff} = 6Ar^{-4} - 2Br^{-3} = \frac{2}{r^3}\left(\frac{A}{r} - B\right)$$

$$W''_{eff}(r_0) = \frac{B^3}{4A^3}\left(\frac{B}{2} - B\right) = \frac{-B^4}{8A^3}$$

zde mi vyšlo záporné znaménko, má být ale kladné

$$A = \frac{1}{2} m^{-1} b^2 \qquad B = G m_s m$$

$$k = W'_{eff}(r_0) = \frac{G^4 m_s^4 m^7}{\cancel{8}\,\frac{1}{\cancel{8}}\, b^6} =$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{G^2 m_s^2 m^3}{b^3} \qquad T = \frac{2\pi}{\omega} = \frac{2\pi b^3}{G^4 m_s^2 m^3}$$

Numericky vyjde perioda 1 rok.