Norse Code

by Greg Van Eekhout (Goodreads Author)

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Is this Ragnarok, or just California?

The NorseCODE genome project was designed to identify descendants of Odin. What it found was Kathy Castillo, a murdered MBA student brought back from the dead to serve as a valkyrie in the Norse god's army. Given a sword and a new name, Mist's job is to recruit soldiers for the war between the gods at the end of the world—and to kill th ...more

Paperback, 292 pages

Published May 19th 2009 by Spectra (first published January 1st 2009)

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NORSE

A non-linear relativistic solver for runaway dynamics

Adam Stahl

Matt Landreman, Ola Embréus, Tünde Fülöp

4th Runaway Electron Meeting

Pertuis, France

2016-06-08

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Motivation

- The more runaways, the bigger the problem
- Existing tools break down when more than a few % runaways
- Such RE densities obtainable in experiments
- Relativistic effects are not always taken into account properly

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- The more runaways, the bigger the problem
- Existing tools break down when more than a few % runaways
- Such RE densities obtainable in experiments
- Relativistic effects are not always taken into account properly

- Who says the tools are correct, anyway?
- How does a multi-MeV tail actually affect the rest of the distribution?

One obvious solution...

A non-linear solver!

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Oh...and make it fully relativistic

A non-linear solver!

Oh...and make it fully relativistic

- 2D in momentum space, no spatial dependence
- Full Braams & Karney collision operator
- Arbitrary electric field strengths
- Synchrotron radiation reaction
- Time-dependent plasma parameters

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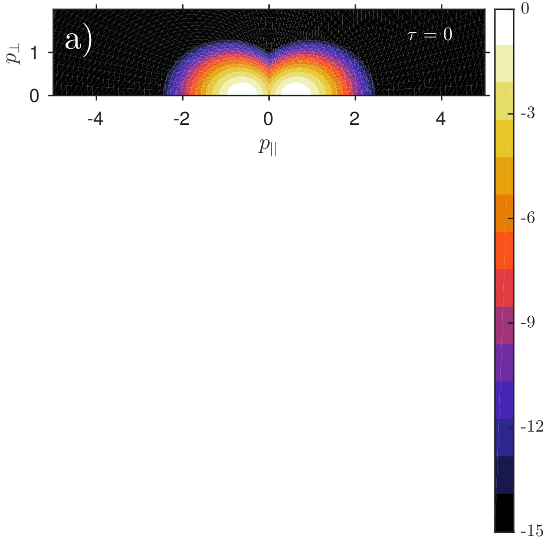
Generation mechanisms:

- Dreicer
- Hot-tail
- Avalanche

Proof of principle



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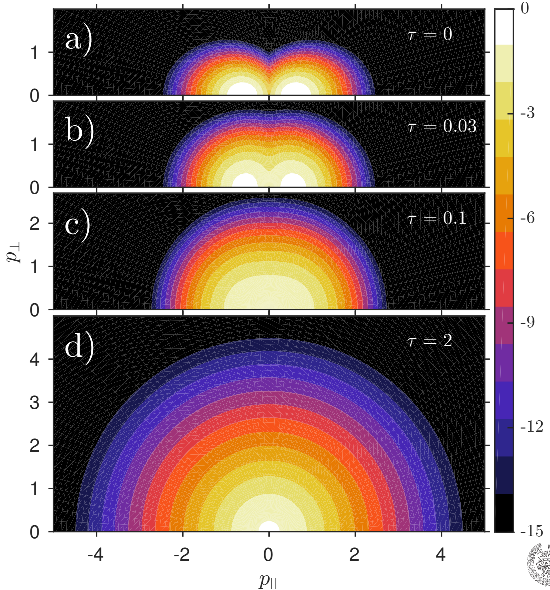

Complicating factors

How does one define a momentum-space runaway region when the bulk is shifting?

Is the runaway concept even meaningful for strong fields?

Is the avalanche growth rate affected by a moving (or even just hot) bulk?

Outline

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Non-linearity: the e-e collision operator [Braams & Karney, PoF B 1, 1355 (1989)]

$$\frac{\partial f}{\partial t} - \frac{e\mathbf{E}}{m_e c} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_s f) = C_{ee}\{f\} + C_{ei}\{f\} + S$$

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- Non-linear because
  - $\mathbb{D}$ and $\mathbf{F}$ depend on potentials $\Upsilon_-\{f\}$, $\Upsilon_+\{f\}$ and $\Pi\{f\}$
  - these depend on the distribution

$$C_{ee}\{f\} = \alpha \frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbb{D} \cdot \frac{\partial f}{\partial \mathbf{p}} - \mathbf{F} f \right)$$

$$\mathbb{D} = \gamma^{-1} \left[\mathbb{L}\Upsilon_- - (\mathbb{I} + \mathbf{pp})\Upsilon_+ \right]$$

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$$\mathbf{F} = \gamma^{-1}\mathbf{K}\Pi$$

- Linearly implicit time advance
  - Potentials from current distribution
  - Normal linear system
  - Time step needs to be reasonably short

- Direct or iterative solver, adaptive time step

Numerical scheme

- Matlab (object oriented)
- Non-uniform 2D finite-difference grid (p, ξ)
- Finite-difference–Legendre-mode representation for calculating potentials
- Efficient mapping between these

NORSE

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- Non-uniform 2D finite-difference grid (p,ξ)
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NORSE

- Nice conservation properties
- Efficient (mostly matrix operations)

Outline

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Benchmark: relativistic weak-field conductivity

- Braams & Karney list conductivities
    - weak-field
    - large T range
    - same collision operator

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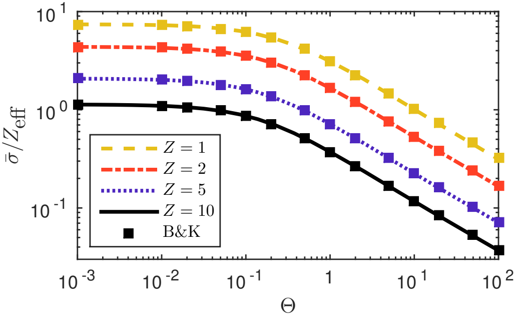
Benchmark: relativistic weak-field conductivity

- Braams & Karney list conductivities
  - weak-field
  - large T range
  - same collision operator
- NORSE reproduces these perfectly



$\bar{\sigma}$: normalized conductivity

$\Theta = T/m_e c^2$

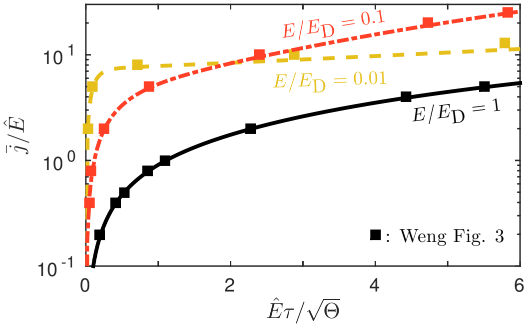
Outline

1. The concept
2. The implementation
3. Relativistic conductivity
4. **Non-linear effects**
5. Summary

Benchmark: conductivity in strong fields

- Comparison to Weng et al. [PRL **100**, 185001 (2008)]
- Strong fields, but
- Non-relativistic
- Nice agreement!

(Probably numerical heating in Weng's data for $E/E_{\mathrm{D}} = 0.01$)



$\bar{j}/\hat{E}$: normalized conductivity
$\hat{E}\tau/\sqrt{\Theta}$: normalized time

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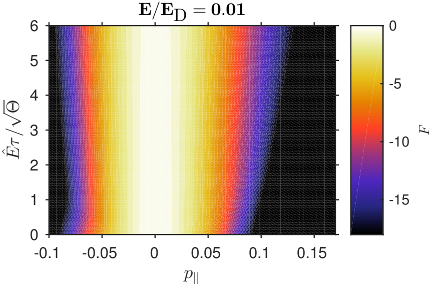
Distribution evolution



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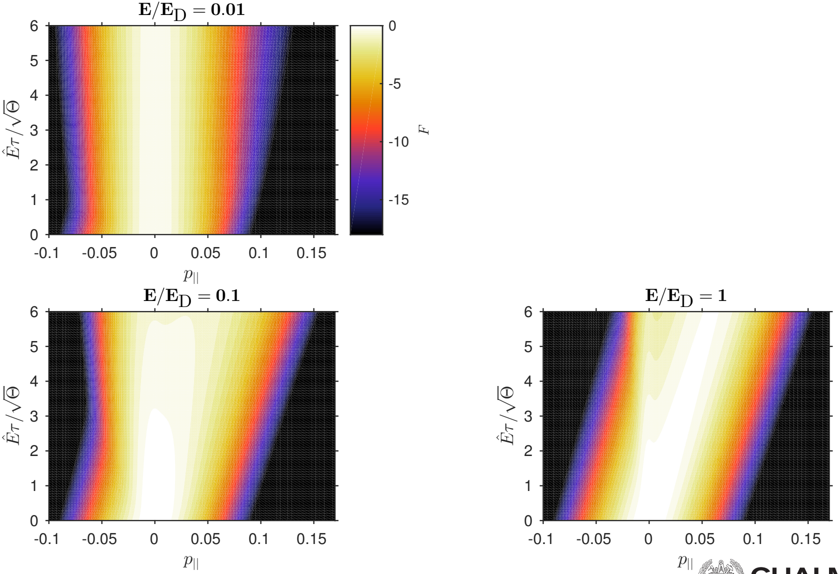
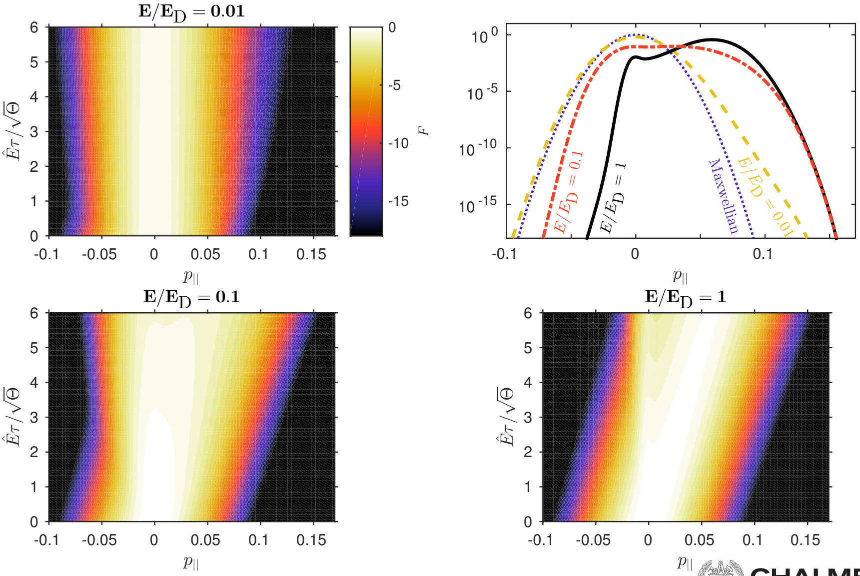


Distribution evolution



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Bulk heating

- E field is a source of heat!
  - Must be removed in a linear treatment
  - Automatically accounted for in NORSE
- In practice bulk keeps temperature or even cools – a heat sink is useful

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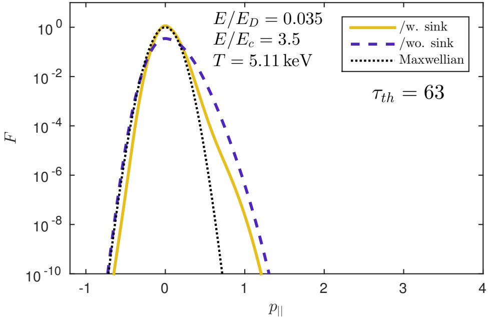

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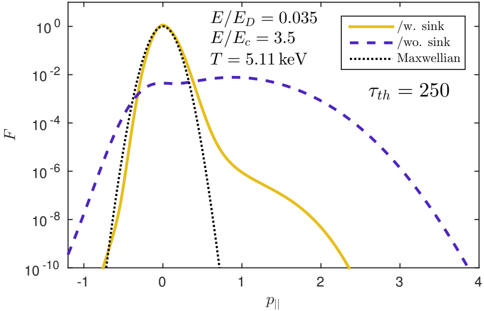

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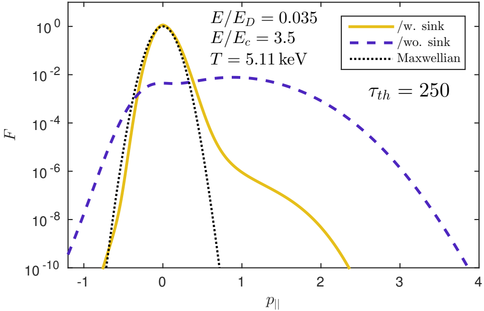

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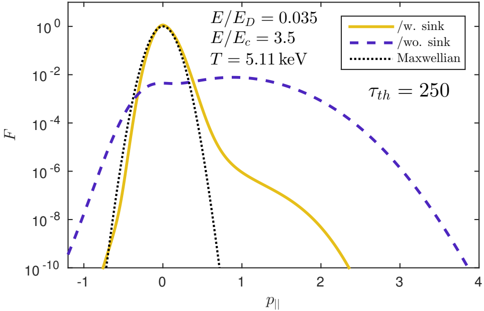
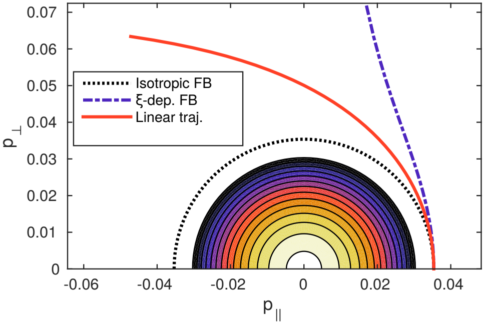


Yes, at least if it is "wide" enough

Runaway region

- Analytic expressions for the separatrix assume Maxwellian bulk
- What to do when distribution can be arbitrary?



Runaway region

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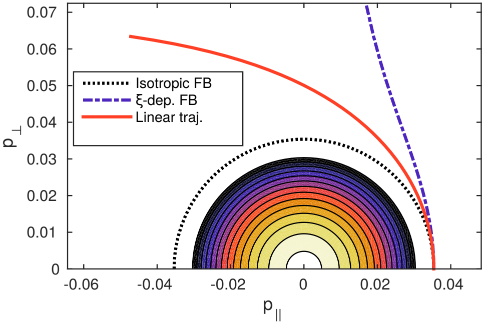
- Consider force balance with friction force taken from f

$$\frac{\mathrm{d}p}{\mathrm{d}t} = F^p_{\mathrm{E}} - F^p_{C_{ee}} - F^p_{\mathrm{S}}$$
$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = F^\xi_{\mathrm{E}} - F^\xi_{C_{ee}} - F^\xi_{\mathrm{S}}$$

- Integrate $\mathrm{d}p/\mathrm{d}\xi$ numerically



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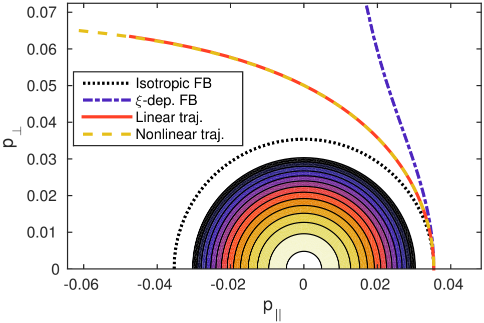

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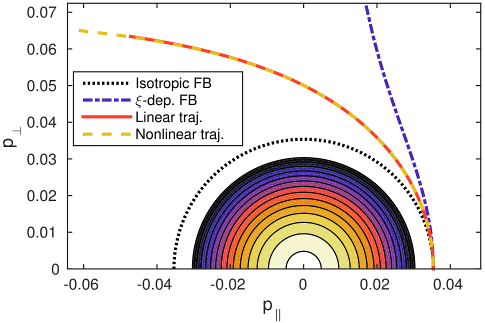

Great! Let's calculate the RE growth rate for high fields!

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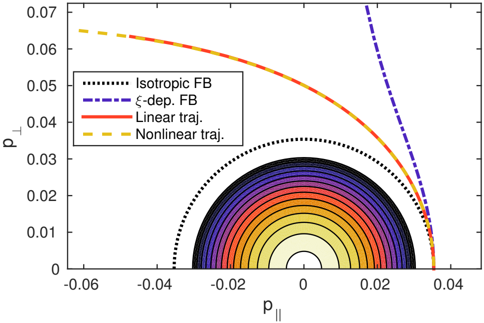

Great! Let's calculate the RE growth rate for high fields!

Wait...not so fast!

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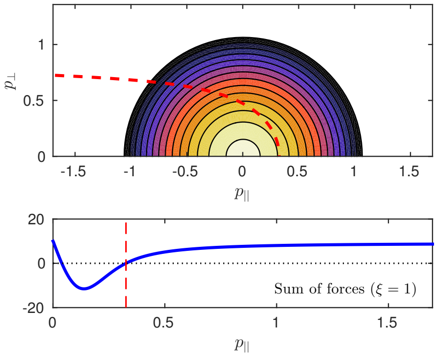
What is a runaway, anyway?

- As bulk distribution smears, collisional friction reduces
- Force balance is shifted



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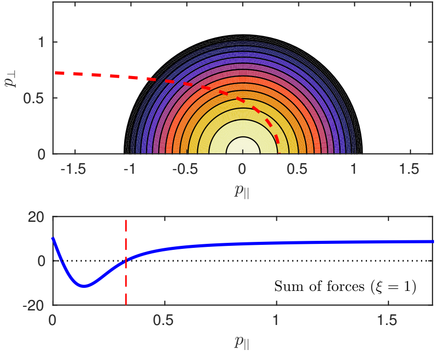
What is a runaway, anyway?

- As bulk distribution smears, collisional friction reduces
- Force balance is shifted
- Eventually sum of forces positive everywhere
  - slide away
  - everything is in the runaway region
  - in essence: "effective" E_D is lowered by the distortion of the distribution



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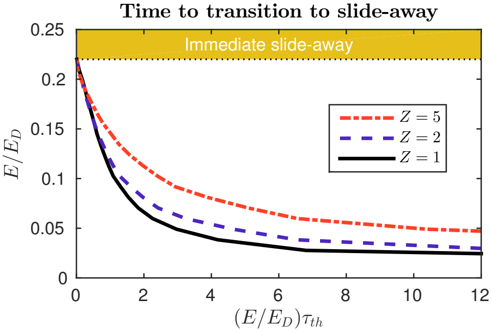

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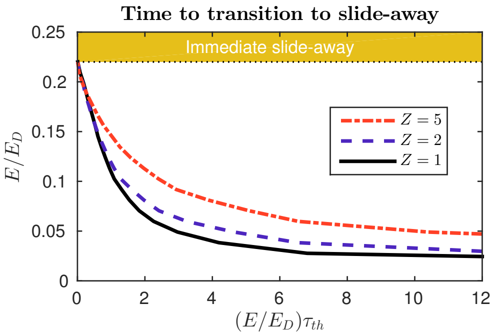
  - also for weak fields!



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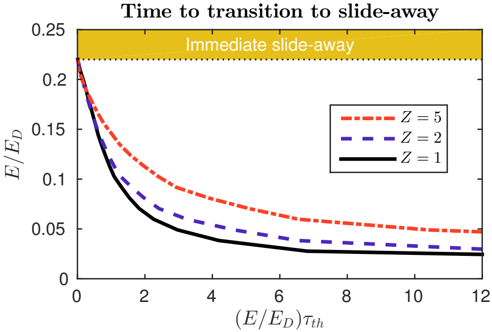
- Is the runaway concept even meaningful?



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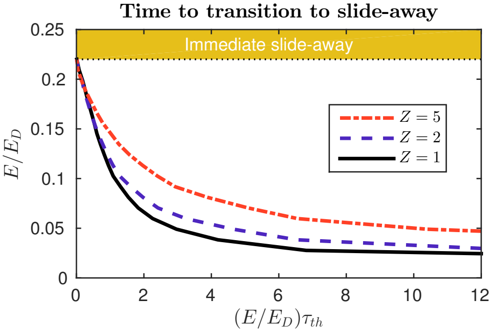


- If bulk temperature kept constant:
  - Weak fields: No (or significantly delayed) transition to slide-away
  - Strong fields: not possible numerically to remove heat

Outline

1. The concept
2. The implementation
3. Relativistic conductivity
4. Non-linear effects
5. **Summary**

Summary

NORSE

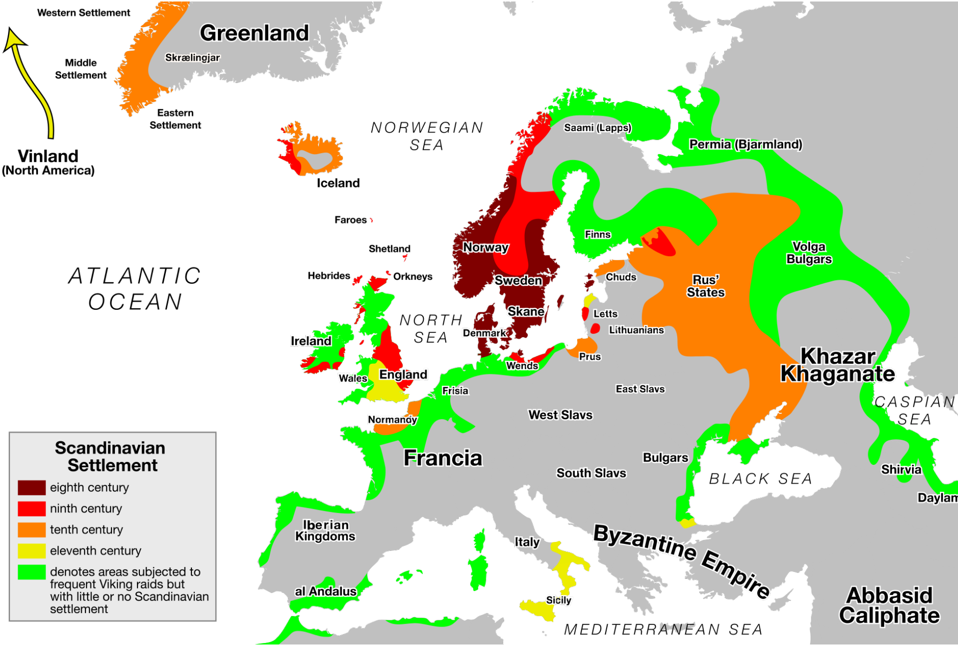
- Relativistic, non-linear electron dynamics
- Radiative effects, time-dependent scenarios
- Efficient, freely available (soon)

Non-linear effects

- Large heating of bulk
- Dynamic runaway region must be used
- Distortion of distribution lowers E_{D}

Outlook

- A few things left to add/polish
- Further investigations to come
- Feel free to use it!


Western Settlement
Greenland
Middle Settlement
Skrælingjar
Eastern Settlement
Vinland
(North America)
NORWEGIAN SEA
Saami (Lapps)
Permia (Bjarmland)
Iceland
Faroes
Finns
Shetland
Norway
Volga Bulgars
ATLANTIC OCEAN
Hebrides
Orkneys
Sweden
Chuds
Rus' States
Skane
NORTH SEA
Letts
Denmark
Lithuanians
Ireland
Khazar Khaganate
Prus
Wends
Wales
England
Frisia
East Slavs
CASPIAN SEA
West Slavs
Normandy
Bulgars
Francia
South Slavs
Shirvia
BLACK SEA
Daylam
Iberian Kingdoms
Italy
Byzantine Empire
al Andalus
Sicily
Abbasid Caliphate
MEDITERRANEAN SEA
Scandinavian Settlement
eighth century
ninth century
tenth century
eleventh century
denotes areas subjected to frequent Viking raids but with little or no Scandinavian settlement

Spare slides

Kinetic equation

$$\frac{\partial f}{\partial t} - \frac{e\mathbf{E}}{m_\mathrm{e} c} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_\mathrm{S} f) = C\{f\} + S$$

- F_S: synchrotron radiation-reaction losses
- C_ee: fully relativistic, non-linear Braams & Karney operator with 5 relativistic potentials
- C_ei: simple, stationary ion model (pitch-angle scattering only)
- C_B: bremsstrahlung collisional losses
- S: knock-on, heat and particle sources

Electron-electron collision operator

$$C_{ee}\{f\} = \alpha \frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbb{D} \cdot \frac{\partial f}{\partial \mathbf{p}} - \mathbf{F} f \right)$$

[Braams & Karney, PoF B 1, 1355 (1989)]

$$\mathbb{D} = \gamma^{-1} \left[\mathbb{L}\Upsilon_- - (\mathbb{I} + \mathbf{pp})\Upsilon_+ \right] \qquad \mathbf{F} = \gamma^{-1}\mathbf{K}\Pi$$

$$\mathbb{L}\Upsilon_- = (\mathbb{I} + \mathbf{pp}) \cdot \frac{\partial^2 \Upsilon_-}{\partial \mathbf{p} \partial \mathbf{p}} \cdot (\mathbb{I} + \mathbf{pp}) + (\mathbb{I} + \mathbf{pp}) \left(\mathbf{p} \cdot \frac{\partial \Upsilon_-}{\partial \mathbf{p}} \right) \qquad \mathbf{K}\Pi = (\mathbb{I} + \mathbf{pp}) \cdot \frac{\partial \Pi}{\partial \mathbf{p}}.$$

$$\Upsilon_- = 4\Upsilon_2 - \Upsilon_1 \qquad \Upsilon_+ = 4\Upsilon_2 + \Upsilon_1 \qquad \Pi = 2\Pi_1 - \Pi_0$$

$$L_0\Upsilon_0 = f, \quad L_2\Upsilon_1 = \Upsilon_0, \quad L_2\Upsilon_2 = \Upsilon_1, \quad L_1\Pi_0 = f, \quad L_1\Pi_1 = \Pi_0$$

$$L_a\Psi = (\mathbb{I} + \mathbf{pp}) : \frac{\partial^2 \Psi}{\partial \mathbf{p} \partial \mathbf{p}} + 3\mathbf{p} \cdot \frac{\partial \Psi}{\partial \mathbf{p}} + \left(1 - a^2\right) \Psi$$

Electron-electron collision operator

$$\frac{C_{ee}\{f\}}{\alpha} = W^{p^2}\frac{\partial^2 f}{\partial p^2} + W^{p}\frac{\partial f}{\partial p} + W^{\xi^2}\frac{\partial^2 f}{\partial \xi^2} + W^{\xi}\frac{\partial f}{\partial \xi} + W^{p\xi}\frac{\partial^2 f}{\partial p \partial \xi} + W^{f} f$$

$$\begin{aligned}
W^{p^2} &= \gamma(8\Upsilon_2 - \Upsilon_0) - 2\frac{\gamma^3}{p}\frac{\partial \Upsilon_-}{\partial p} - \frac{\gamma(1-\xi^2)}{p^2}\frac{\partial^2 \Upsilon_-}{\partial \xi^2} + 2\frac{\gamma\xi}{p^2}\frac{\partial \Upsilon_-}{\partial \xi}, \\
W^{p} &= \frac{1}{\gamma p}(2+3p^2)(8\Upsilon_2 - \Upsilon_0) - 16\gamma\frac{\partial \Upsilon_2}{\partial p} + 6\gamma\frac{\partial \Upsilon_1}{\partial p} - \gamma\frac{\partial \Upsilon_0}{\partial p} - 2\frac{\gamma^3}{p}\left(\frac{\partial^2 \Upsilon_-}{\partial p^2} + \frac{1}{p}\frac{\partial \Upsilon_-}{\partial p}\right) \\
&\quad + \frac{1}{\gamma p}\left(2 + \frac{1}{p^2}\right)\left(2\xi\frac{\partial \Upsilon_-}{\partial \xi} - (1-\xi^2)\frac{\partial^2 \Upsilon_-}{\partial \xi^2}\right) - \gamma\frac{\partial \Pi}{\partial p}, \\
W^{\xi^2} &= \frac{1-\xi^2}{\gamma p^2}\left(\frac{\gamma^2}{p}\frac{\partial \Upsilon_-}{\partial p} + \frac{1}{p^2}\left[(1-\xi^2)\frac{\partial^2 \Upsilon_-}{\partial \xi^2} - \xi\frac{\partial \Upsilon_-}{\partial \xi}\right] - \Upsilon_+\right), \\
W^{\xi} &= -\frac{\xi(1-\xi^2)}{\gamma p^4}\frac{\partial^2 \Upsilon_-}{\partial \xi^2} - 2\frac{\gamma(1-\xi^2)}{p^3}\frac{\partial^2 \Upsilon_-}{\partial p \partial \xi} - 2\frac{\gamma\xi}{p^3}\frac{\partial \Upsilon_-}{\partial p} \\
&\quad + \left(\frac{2}{\gamma p^4} + 3\frac{1-\xi^2}{\gamma p^2}\right)\frac{\partial \Upsilon_-}{\partial \xi} - \frac{1-\xi^2}{\gamma p^2}\left(4\frac{\partial \Upsilon_2}{\partial \xi} - 3\frac{\partial \Upsilon_1}{\partial \xi} + \frac{\partial \Upsilon_0}{\partial \xi} + \frac{\partial \Pi}{\partial \xi}\right) + 2\frac{\xi}{\gamma p^2}\Upsilon_+, \\
W^{p\xi} &= 2\frac{\gamma(1-\xi^2)}{p^3}\left[p\frac{\partial^2 \Upsilon_-}{\partial p \partial \xi} - \frac{\partial \Upsilon_-}{\partial \xi}\right], \\
W^{f} &= -\gamma\frac{\partial^2 \Pi}{\partial p^2} - \frac{1}{\gamma p}\left(2+3p^2\right)\frac{\partial \Pi}{\partial p} - \frac{1-\xi^2}{\gamma p^2}\frac{\partial^2 \Pi}{\partial \xi^2} + 2\frac{\xi}{\gamma p^2}\frac{\partial \Pi}{\partial \xi}.
\end{aligned}$$

What about avalanche?

Avalanche operators assume cold bulks

- Is the avalanche growth rate affected by a moving (or even just hot) bulk?
- Avalanche not important for very high fields, but
- Distribution is still eventually distorted, even at low fields

Future work!