Effect of bremsstrahlung emission on runaway electrons

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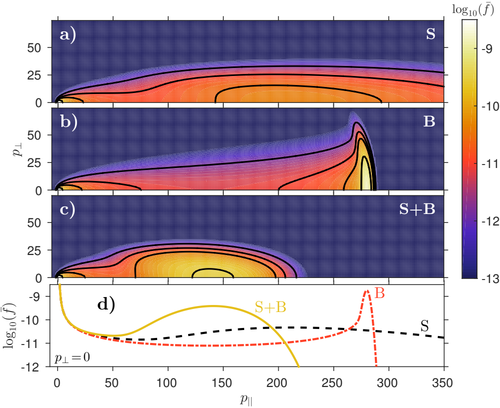
Last time on *REM in Pertuis*

What we presented last year:

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{brems}} = -\frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbf{F}_{\text{brems}} f_e\right)$$

$$F_{\text{brems}} = -n_e \int k \frac{\partial \sigma}{\partial k}\,\mathrm{d}k$$

Bremsstrahlung is more important than synchrotron when $n_e[10^{20}\mathrm{m}^{-3}] \gtrsim B[T]^2$.

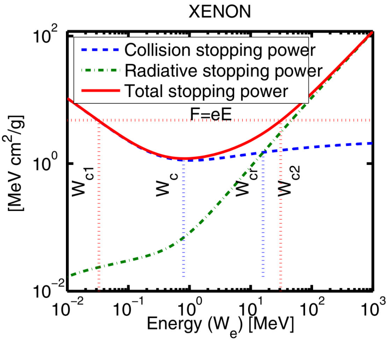


Previous work

PRL **94,** 215003 (2005) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2005

Role of Bremsstrahlung Radiation in Limiting the Energy of Runaway Electrons in Tokamaks

M. Bakhtiari,[1,*] G. J. Kramer,[2] M. Takechi,[1] H. Tamai,[1] Y. Miura,[1] Y. Kusama,[1] and Y. Kamada[1]



In summary, by using the collision and radiative stopping powers of different noble gases we have shown that bremsstrahlung radiation can be an effective energy limit for runaway electrons in Tokamaks.

Previous work

AIP | Physics of Plasmas

#1 JOURNAL IN 2014 GOOGLE SCHOLAR METRICS FOR THE PLASMA & FUSION CATEGORY

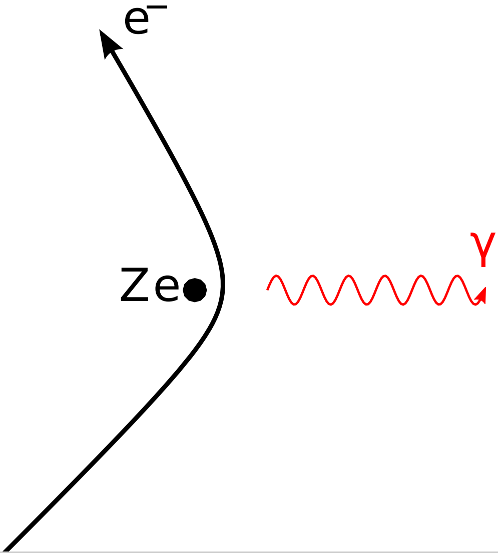
Determination of the parametric region in which runaway electron energy losses are dominated by bremsstrahlung radiation in tokamaks

I. Fernández-Gómez, J. R. Martín-Solís, and R. Sánchez

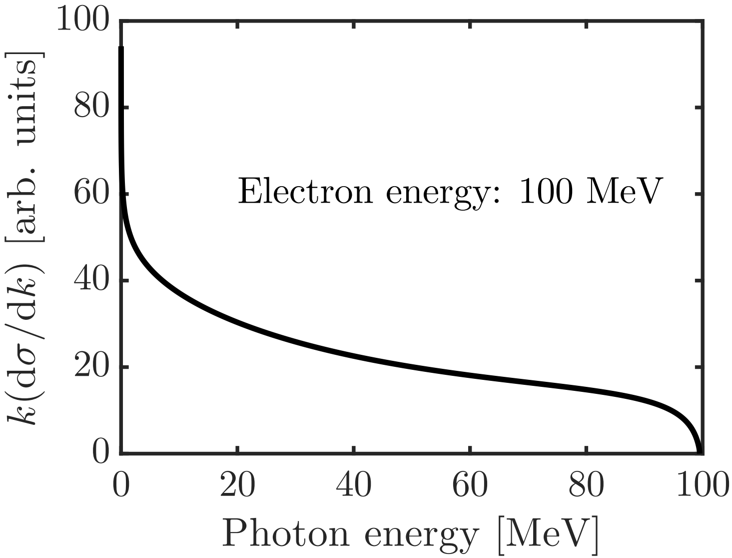
Citation: Physics of Plasmas (1994-present) **14**, 072503 (2007); doi: 10.1063/1.2746219

"The effect of bremsstrahlung, although overestimated in [Bakhtiari *et al.*], has been shown to be still large enough to be able to greatly reduce the maximum attainable runaway energy for high enough values of the effective charge and electron density."

Modelling bremsstrahlung losses



Modelling bremsstrahlung losses

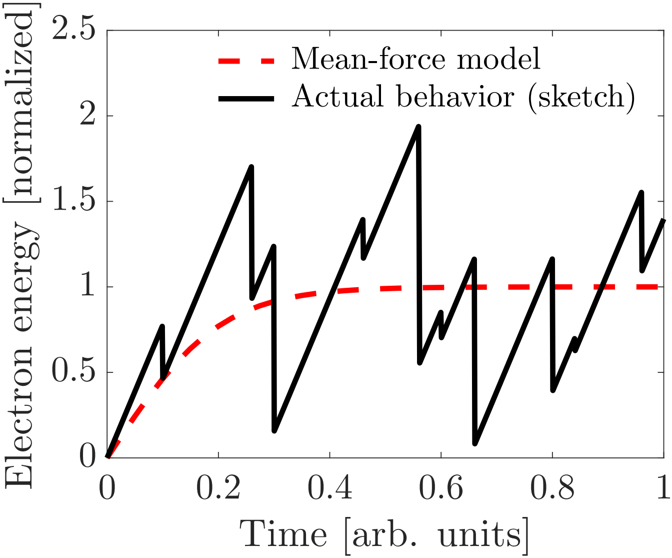


Modelling bremsstrahlung losses

Two options:

1. Account for the average energy loss with a stopping-power formula
2. Fully treat the stochastic nature of the radiation

Modelling bremsstrahlung losses



Modelling bremsstrahlung losses

Effect on a distribution of electrons?

1. Mean-force model:

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{brems}} = -\frac{\partial}{\partial \mathbf{p}} \cdot \Big(\mathbf{F}_{\text{brems}}(\mathbf{p}) f_e(\mathbf{p})\Big)$$

2. Physically accurate model (Boltzmann operator):

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{brems}} = C^{\text{brems}}(\mathbf{p}) = \sum_b \Bigg\{ \int \mathrm{d}\mathbf{p}_1\, f_e(\mathbf{p}_1) \int \mathrm{d}\mathbf{p}_2\, \bar{v}_{\text{rel}} f_b(\mathbf{p}_2) \frac{\partial \bar{\sigma}_{eb}}{\partial \mathbf{p}} - f_e(\mathbf{p}) \int \mathrm{d}\mathbf{p}'\, v_{\text{rel}} f_b(\mathbf{p}')\, \sigma_{eb} \Bigg\}$$

Modelling bremsstrahlung losses

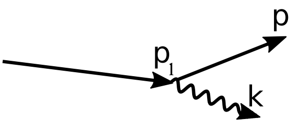
Targets are stationary (compared to multi-MeV runaways):

$$C^{\mathsf{brems}} = (1+Z_{\mathsf{eff}})n_e\left(\int \mathrm{d}\mathbf{p}_1\, v_1 f_e(\mathbf{p}_1)\frac{\partial\bar{\sigma}_0}{\partial\mathbf{p}} - v f_e(\mathbf{p})\sigma_0(\mathbf{p})\right)$$

The formula for $\frac{\partial\bar{\sigma}}{\partial\mathbf{p}}$ is given in
[G. Racah. Il Nuovo Cimento **11**, 461 (1934)]
[P. T. McCormick, D. G. Keiffer and G. Parzen. Phys. Rev. **103**, 29 (1956)] [fixed a misprint in the 1934 Racah formula]

Modelling bremsstrahlung losses

$$\frac{\partial\bar{\sigma}}{\partial p} = \int$$



$$dk$$

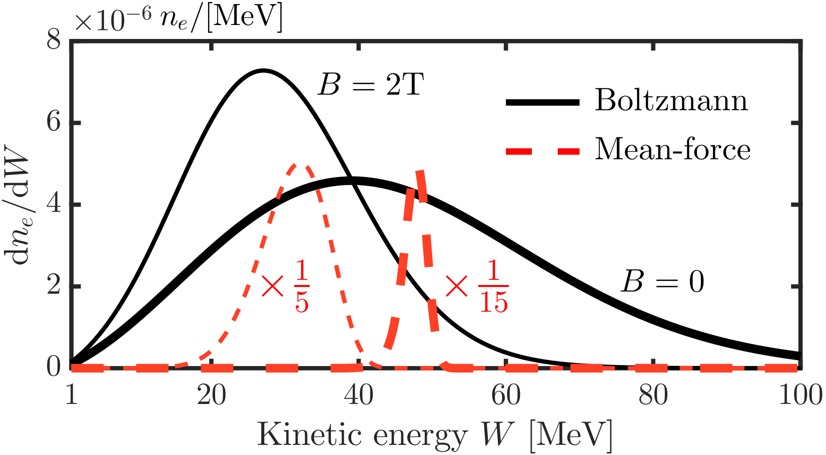
Results

The scenario we consider:

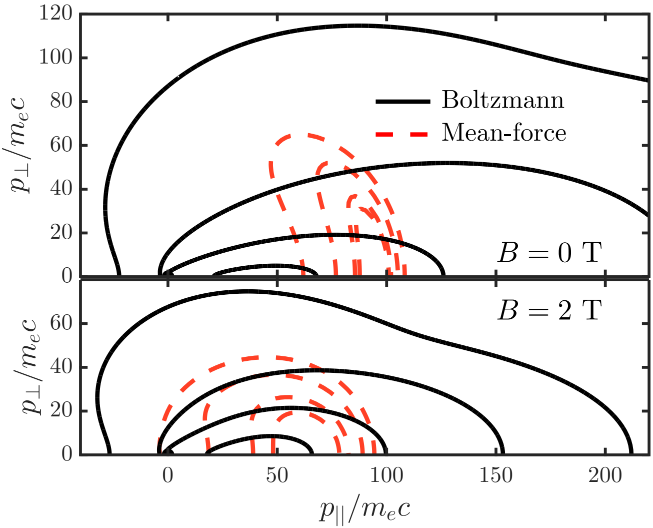
- Post-disruption with gas injection (high n_e)
- The late stages (low E/E_c, steady-state distribution)

Results

Post-disruption scenario with successful MGI:
Simulations using CODE with $n_e = 3 \cdot 10^{21}$ m^{-3}, $Z_{\text{eff}} = 10$, $E = 2E_c$.

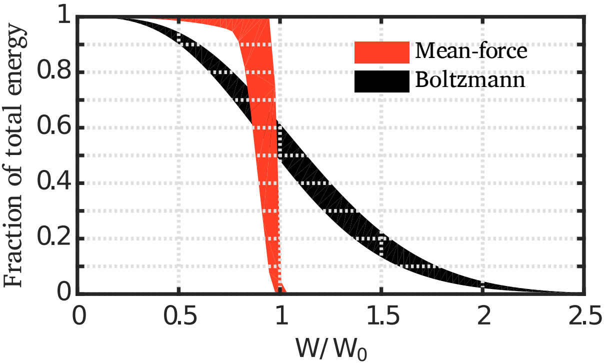


Results


120
100
80
60
40
20
0
$p_\perp/m_ec$
Boltzmann
Mean-force
$B = 0$ T
$B = 2$ T
60
40
20
0
$p_\perp/m_ec$
0
50
100
150
200
$p_{||}/m_ec$

Results

Shape of distribution is almost independent of plasma parameters! (when rescaled)



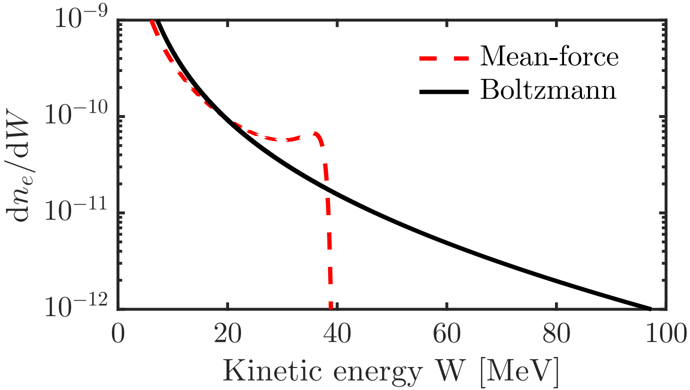
$$Z_{\text{eff}} \in [1, 35]$$

$$(E/E_c - 1)/(Z_{\text{eff}} + 1) \in [0.05, 0.25]$$

[O. Embréus, A. Stahl and T. Fülöp. Submitted for publication to New Journal of Physics]

Results

What about with avalanche?



The same conclusions hold! (approximately)

Conclusions (1 of 2)

- Bremsstrahlung is important in limiting the maximum runaway energy. $n_e \gtrsim B^2$
- The random nature of bremsstrahlung emission matters
- Stopping-power estimates give *average* energies, not maximum
- In steady-state, $\approx 5\%$ of electrons have more than twice the expected energy (less when synchrotron is accounted for)

But wait, there is more!

$$\frac{\partial\sigma}{\partial\mathbf{p}} \propto \frac{1}{k}$$

The cross-section diverges for small photon energies – how to deal with?

Low-energy photon contribution

First attempt:

Cut off stopping-power integral at k_0,

$$F_{k_0} = n_e \int_{k_0}^{\gamma-1} k \frac{\partial \sigma}{\partial k} \mathrm{d}k.$$

The relative error is

$$\frac{F_{k_0} - F_0}{F_0} = \mathcal{O}(k_0/p).$$

$\Rightarrow$ Seems safe to cut off at some small $k_0 \ll p$!

Low-energy photon contribution

However, the transport cross-section

$$\sigma_t = \int (1 - \cos\theta) \frac{\partial\sigma}{\partial k \cos\theta} \,\mathrm{d}k \,\mathrm{d}\!\cos\theta$$

diverges logarithmically.

$\Rightarrow$ Electrons are pitch-angle scattered infinitely strongly?

Low-energy photon contribution

Natural cut-off at $k \sim \hbar\omega_p$, which regularizes σ_t.

A *Bremsstrahlung logarithm* $\ln \Lambda_\mathsf{B} = \ln \frac{k_0}{\hbar\omega_p} \sim 20$ emerges.

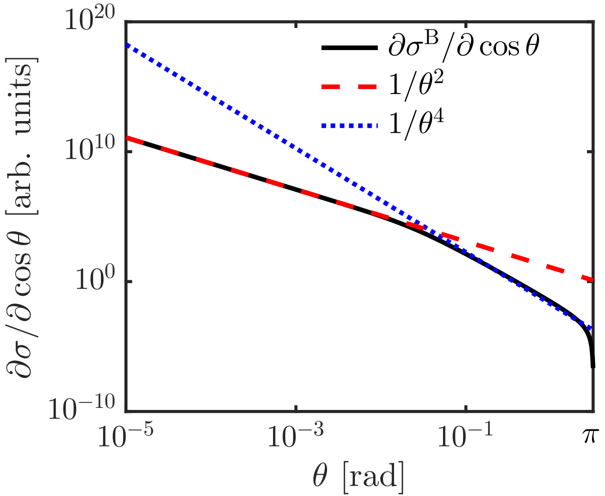
$$\frac{\sigma_t^{\text{brems}}}{\sigma_t^{\text{Coulomb}}} = \frac{4}{137\pi}\frac{\ln \Lambda_\mathsf{B}}{\ln \Lambda}\left[\left(\ln\left(\frac{2p}{m_e c}\right) - 1\right)^2 + 1\right]$$

for $p \gg m_e c$.

$$\Rightarrow \begin{array}{l} \approx 10\% \quad \text{correction at } 30\,\text{MeV} \\ \approx 100\% \text{ correction at } 30\,\text{GeV}. \end{array}$$

Low-energy photon contribution

Curiously, the low-k emission is dominated by large-angle deflections



Conclusions (2 of 2)

- Low-energy photons are not generally ignorable
- Causes pitch-angle scattering (10% of Coulomb scattering at $30\,\mathrm{MeV}$)
- Deflections dominated by large-angle scatterings
- The effect is captured in our Boltzmann bremsstrahlung model