Cherenkov Radiation energy loss of runaway electrons

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Outline

- Introduction to Cherenkov radiation in plasma
  - Lenard-Balescu collision operator

- Cherenkov radiation in unmagnetized plasma
  - Radiative energy loss
  - Correction to Coulomb logarithm

- Cherenkov radiation in magnetized plasma
  - Transit-time magnetic pumping (TTMP) momentum loss

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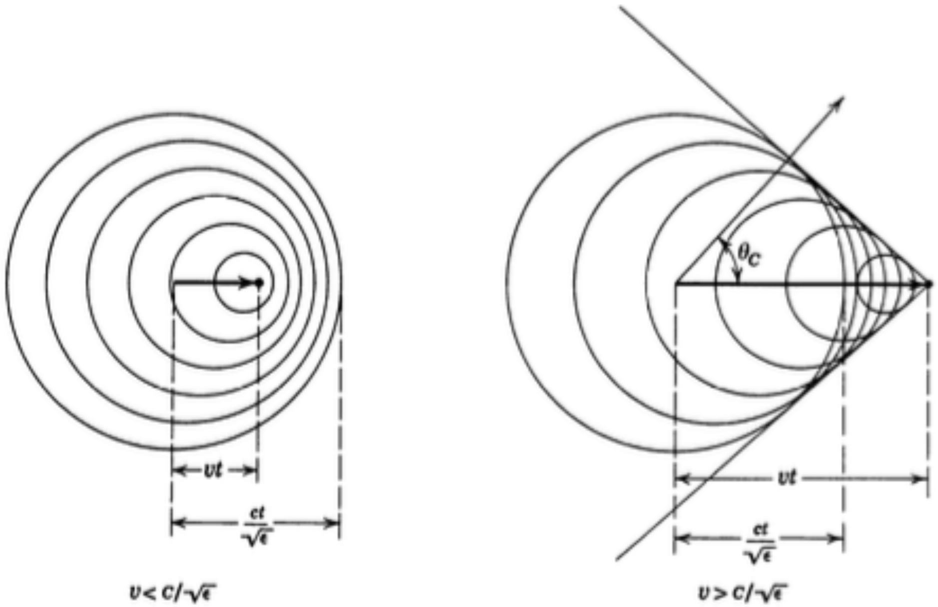
Introduction: Cherenkov radiation

- Cherenkov radiation is emitted when a charged particle passed through a dielectric medium at a speed greater than wave phase velocity in that medium.



- The condition for Cherenkov radiation is $n>1/\beta$ ($n=kc/\omega$, $\beta=v/c$).
- For plasma waves and runaway electrons (β~1), this condition is easy to satisfy.

Introduction: Lenard-Balescu collision operator and Cherenkov radiation in plasmas

$$C[f] = \frac{\partial}{\partial \mathbf{v}} \cdot \left[\left(\frac{q}{m} \right) \mathbf{E}^p f - \mathsf{D} \cdot \frac{\partial f}{\partial \mathbf{v}} \right]$$

$\mathbf{E}^p$: Polarization drag electric field
D: Diffusion from electric field fluctuation

Test particle superposition principle

$$\mathsf{D} = \frac{1}{2} \left(\frac{q}{m} \right)^2 \int d\mathbf{k} \langle \delta \mathbf{E} \delta \mathbf{E} \rangle (\mathbf{k}, \omega = \mathbf{k} \cdot \mathbf{v})$$

- The polarization drag originates from the polarization of the plasma medium due to the electric field of a test particle, which is related to the Cherenkov radiation energy loss.
  - For $\omega/k \sim v_{\mathrm{th}}$, the excited mode is strongly Landau damped and the energy is absorbed by bulk electrons.
  - For $\omega/k \gg v_{\mathrm{th}}$, the mode is weakly damped (mainly through collisions), and energy gets radiated away.

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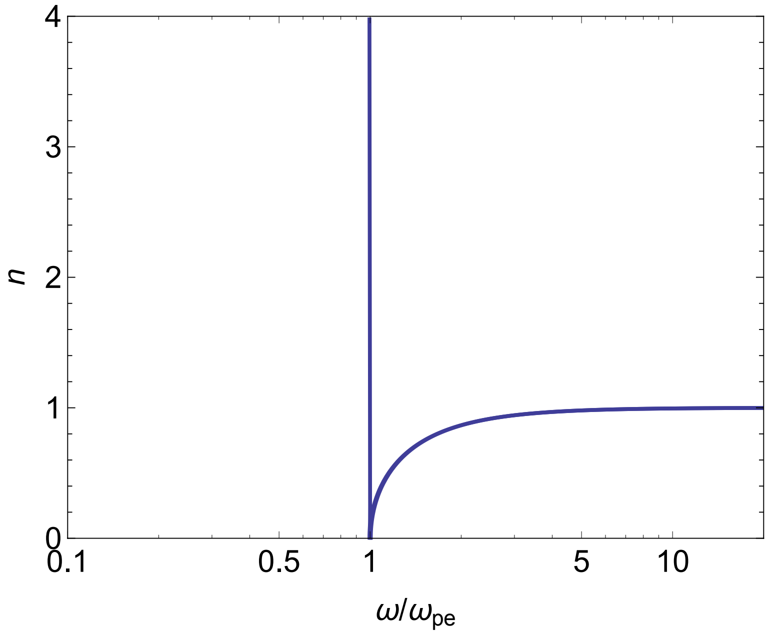
Electron waves in unmagnetized plasma

- In unmagnetized plasma, the electron waves have two branches (ignoring ion effects and thermal correction)
  - Langmuir wave: $\omega^2 = \omega_p^2$
  - Electronmagnetic wave: $\omega^2 = k^2c^2 + \omega_p^2$



- For Cherenkov radiation, only Langmuir wave is possible (n>1).

The energy loss from Cherenkov radiation

- To solve the energy loss due to Cherenkov radiation, one can calculate the excited electric field from a single moving electron in the medium.

$$\left.\begin{array}{l}\text{Ampere's law}\\ \text{Faraday's law}\end{array}\right\} \rightarrow \mathbf{k}\times\mathbf{k}\times\mathbf{E}+\left(\frac{\omega}{c}\right)^2 \underline{\underline{\epsilon}}\cdot\mathbf{E}=-\frac{4\pi i\omega}{c^2}\mathbf{j}$$

- The current from a single moving electron

Cherenkov resonance

$$\mathbf{j}(\mathbf{x},t)=-e\mathbf{v}\delta(\mathbf{x}-\mathbf{x}_0-\mathbf{v}t)\rightarrow \mathbf{j}(\mathbf{k},\omega)=-e\mathbf{v}\delta(\omega-\mathbf{k}\cdot\mathbf{v})\exp(i\mathbf{k}\cdot\mathbf{x}_0)$$

- The time-averaged electric field felt by the moving electron

$$\mathbf{E}^p=\left\langle \int d^3\mathbf{k}\int d\omega\,\mathbf{E}(\mathbf{k},\omega)\exp\left[-i\mathbf{k}(\mathbf{x}_0+\mathbf{v}t)+i\omega t\right]\right\rangle_t$$

- The energy loss is then $\mathbf{E}^p\cdot\mathbf{j}$.

Cherenkov radiation energy loss in unmagnetized plasma

- For unmagnetized plasma

$$\underline{\underline{\epsilon}} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\underline{\underline{1}} = \epsilon \underline{\underline{1}}$$

- Assume electron is moving along z direction,

$$E_z^p = \int d^3\mathbf{k}\, d\omega \frac{4\pi i\, e v\, c^2 \left(\epsilon - \frac{k_{\parallel}^2 c^2}{\omega^2}\right)}{\omega\epsilon\left(\epsilon - \frac{k^2 c^2}{\omega^2}\right)} \delta(\omega - \mathbf{k}\cdot\mathbf{v}), \quad k_{\parallel} = \frac{\mathbf{k}\cdot\mathbf{v}}{v}$$

- The principal value of the integral is imaginary, which will not contribute to the energy loss.
  - However, the residues (which correspond to the modes that satisfy the Cherenkov radiation condition) will give real contribution and energy loss.

Correction to Coulomb logarithm due to Cherenkov radiation loss

- Using the residue theorem to calculate the integral

$$E_z^p = \frac{e\omega_p^2}{v^2}\int \frac{\sin\theta d\theta}{\cos\theta}\sigma(\theta), \qquad \sigma(\theta) = \begin{cases} 1, & \text{if} \cos\theta > \omega_p / k_{\max} v \\ 0, & \text{if} \cos\theta > \omega_p / k_{\max} v \end{cases}$$

$$= \frac{e\omega_p^2}{v^2}\ln\frac{k_{\max} v}{\omega_p}$$

θ Is the wave emittance angle

- Note that we use the cold plasma approximation, so we can choose $k_{\max}=1/\lambda_D$ to ensure thermal effect is not important.

Correction to lnΛ for the drag force

- Recall that the drag force in the Landau collision operator due to binary collisions can be written as

$$E_{drag} = \frac{e\omega_p^2}{v^2}\ln\Lambda, \quad \Lambda = \frac{b_{max}}{b_{min}}$$

$$b_{max} = \lambda_D, \quad b_{min} = \max\left\{\frac{e_\alpha e_\beta}{m_{\alpha\beta}u^2}, \frac{\hbar}{2m_{\alpha\beta}u}\right\}$$

- Cherenkov radiation energy loss gives a correction to lnΛ

$$\ln\Lambda \rightarrow \ln\Lambda + \ln\left(\frac{v}{v_{\text{th}}}\right)$$

  - In DIII-D QRE experiments, for highly relativistic runaway electrons, this correction is about 20%.
  - For electron starting to run away ($v \ll c$), this correction is small.

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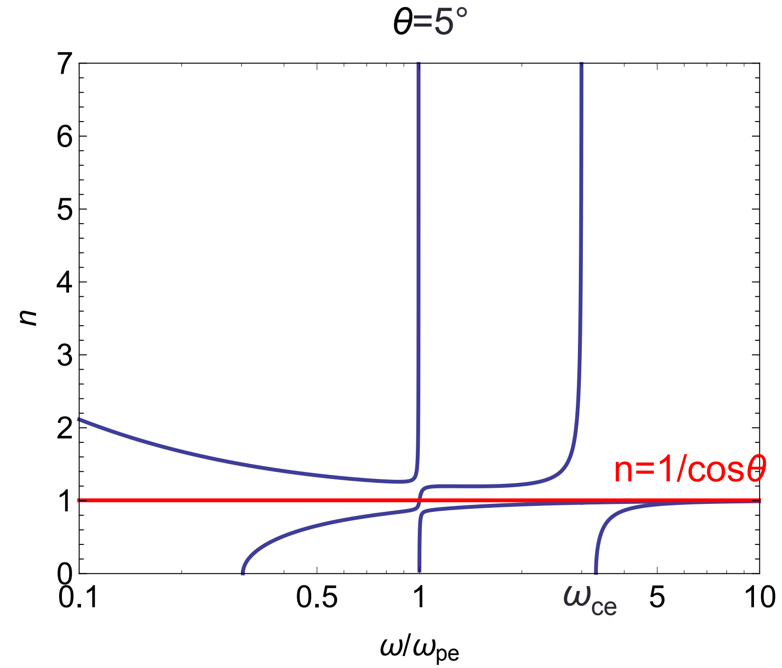
Cherenkov radiation in magnetized plasma

- The dielectric tensor and the dispersion relation in magnetized plasma (ignoring ions and thermal effects)

$$\underline{\underline{\epsilon}} = \begin{pmatrix} 1-\dfrac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} & -i\dfrac{\omega_{ce}}{\omega}\dfrac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} & 0 \\ i\dfrac{\omega_{ce}}{\omega}\dfrac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} & 1-\dfrac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2} & 0 \\ 0 & 0 & 1-\dfrac{\omega_{pe}^2}{\omega^2} \end{pmatrix}$$



- The allowed wave branches for Cherenkov radiation are: Langmuir wave, extraordinary-electron (EXEL) wave, and upper-hybrid wave (electrostatic Bernstein wave).

G.I. Pokol, A. Kómár, A. Budai, A. Stahl, and T. Fülöp, Phys. Plasmas 21, 102503 (2014).

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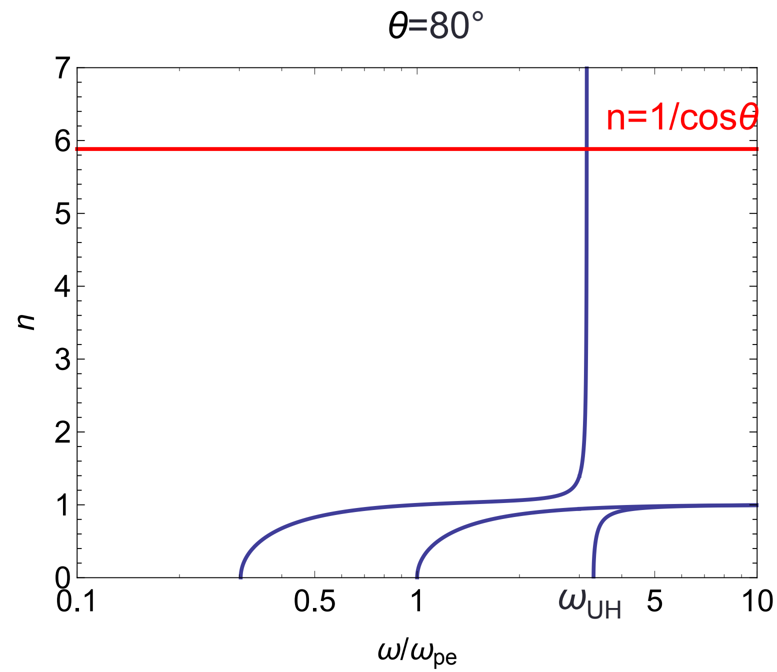

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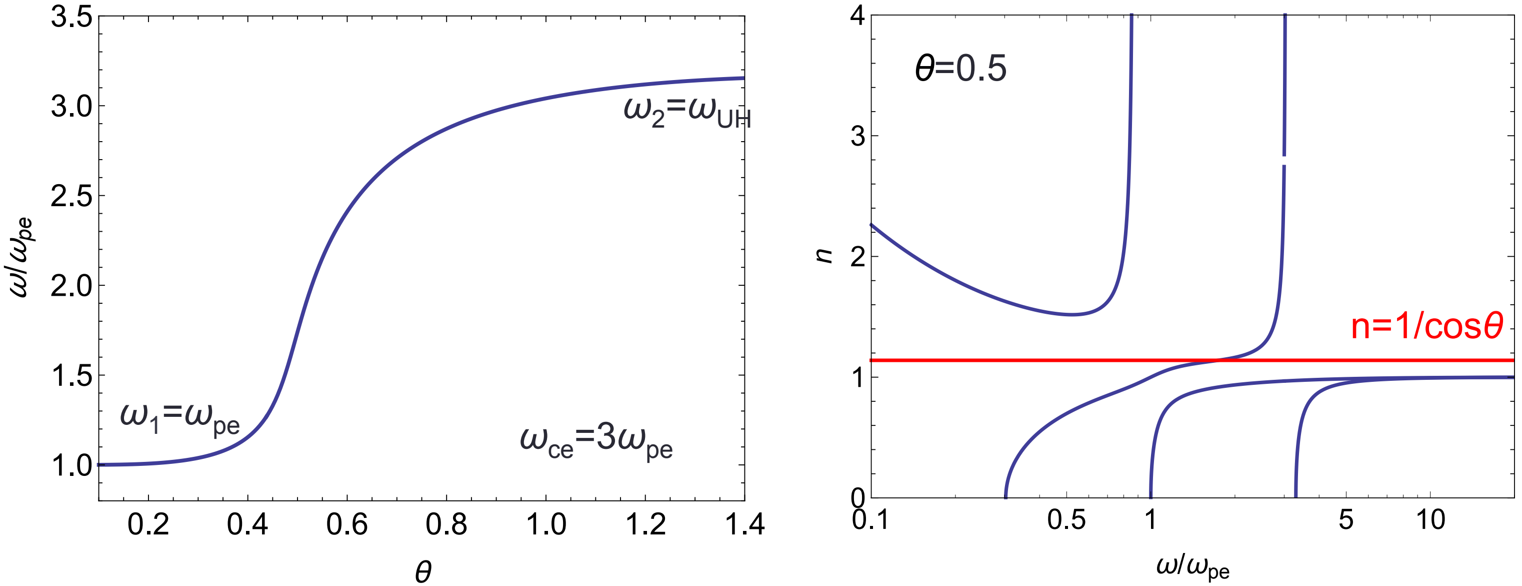
Wave frequency for Cherenkov radiation

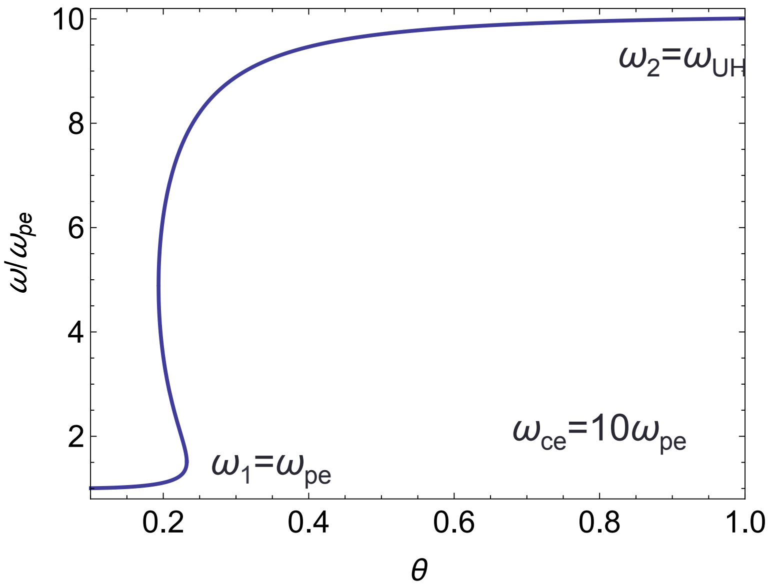
- For a given emittance angle of k, one can calculate the wave frequency that satisfies the Cherenkov radiation condition.

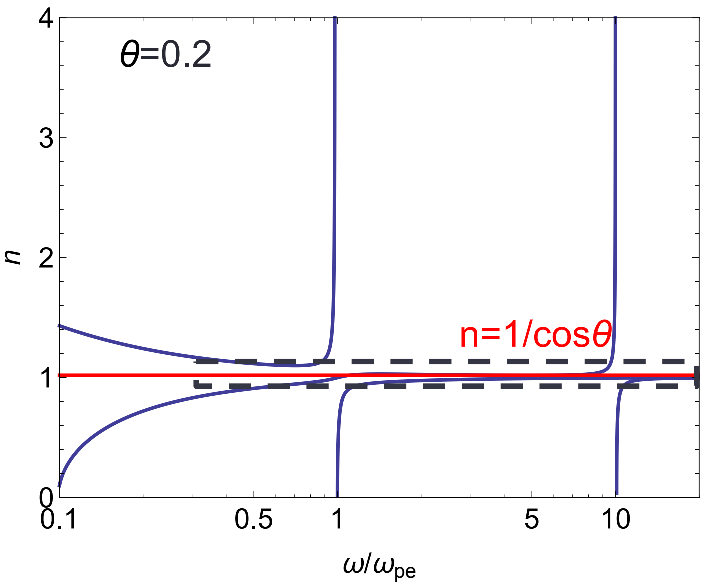


- For ω_{ce}~ω_{pe}, one emittance angle gives one solution of ω, which shifts from ω_{pe} to $\omega_{UH.}$

Wave frequency for Cherenkov radiation (cont'd)

- For a given emittance angle of k, one can calculate the wave frequency that satisfies the Cherenkov radiation condition.





- For $\omega_{ce} \gg \omega_{pe}$, there are 3 branches of ω roots, which all coexist in a range of emittance angle.

Wave frequency for Cherenkov radiation (cont'd)

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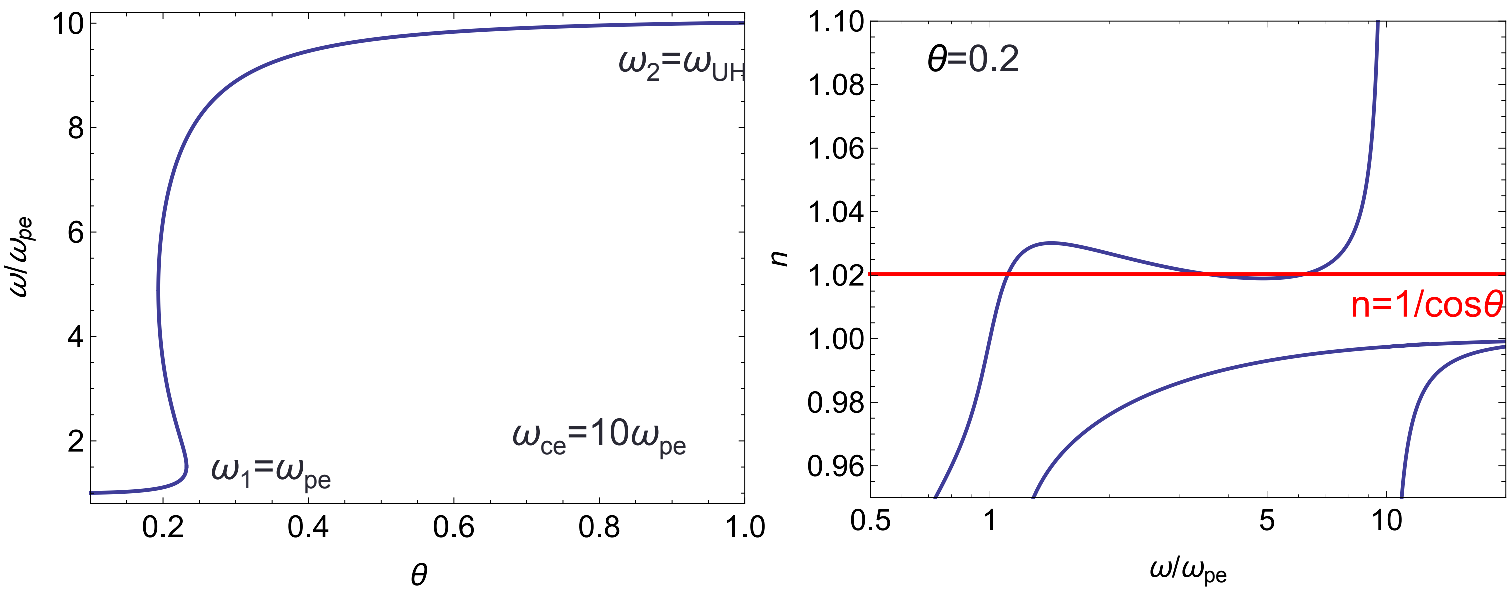
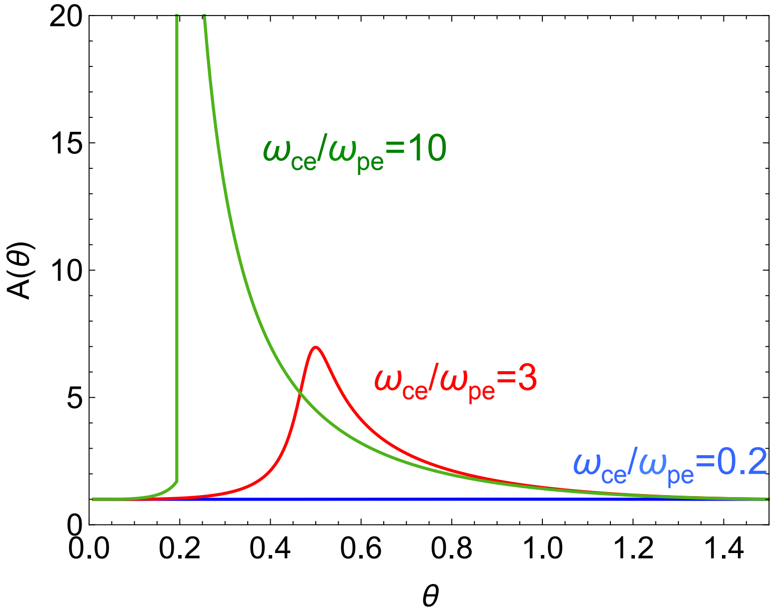


- For $\omega_{ce} \gg \omega_{pe}$, there are 3 branches of ω roots, which all coexist in a range of emittance angle.

Cherenkov radiation energy loss in magnetized plasma

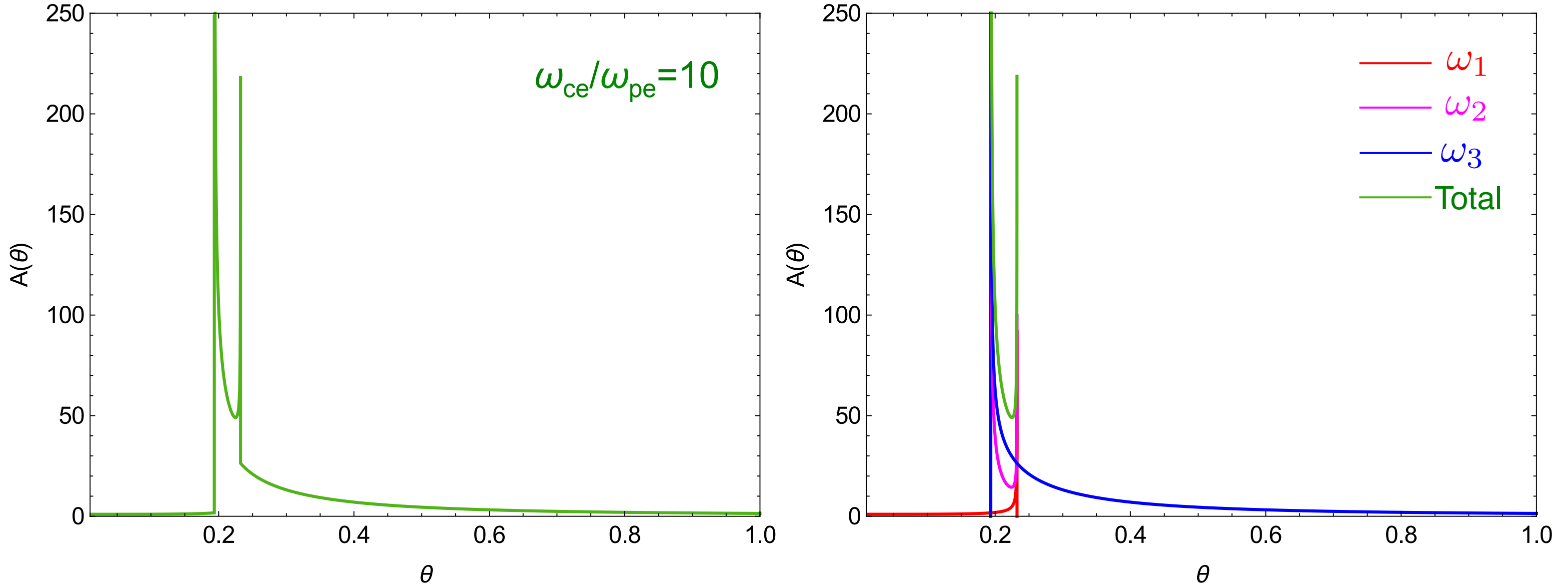
- We can use the same method to calculate the Cherenkov radiation energy loss.
  - Assume that electron is moving perfectly along B field (no $v_\perp$)

$$E_z^p = \frac{e\omega_{pe}^2}{v^2}\int \frac{\sin\theta d\theta}{\cos\theta} A(\theta)$$



- Although the radiation loss from small emittance angle (Langmuir) and large angle (Upper-hybrid) stays the same, intermediate angle (EXEL wave) gives additional energy loss.

Effects of 3 modes on Cerenkov radiation energy loss



- All the 3 modes contributed to the Cherenkov radiation energy loss
- For $\omega_{ce} \gg \omega_{pe}$, the radiation emittance is strongly peaked at certain angle.

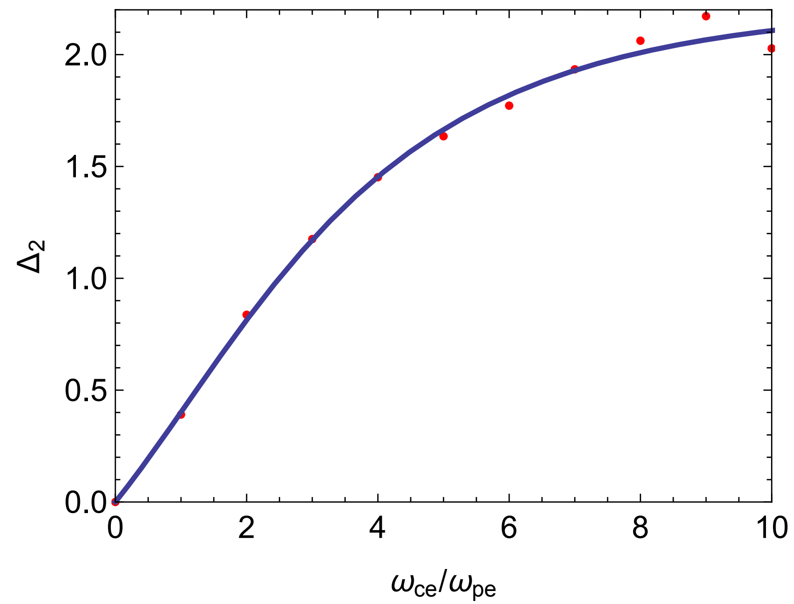
Cherenkov radiation energy loss in magnetized plasma (cont'd)

- The Coulomb logarithm in the drag force has a correction

$$\ln\Lambda \to \ln\Lambda + \ln\left(\frac{v}{v_{\mathrm{th}}}\right) + \Delta_2$$

$$\Delta_2 = \frac{x(a_1 x + a_0)}{(x^2 + b_1 x + b_0)}, \qquad x = \frac{\omega_{ce}}{\omega_{pe}}$$

$$a_1 = 2.05, a_0 = 9.51$$
$$b_1 = 1.63, b_0 = 26.23$$



Transit-time magnetic pumping (TTMP) associated with Cherenkov radiation

- Now we consider runaway electrons with finite $v_{\perp}$.
- In addition to electric field, gradient of magnetic field can also cause momentum loss in parallel direction

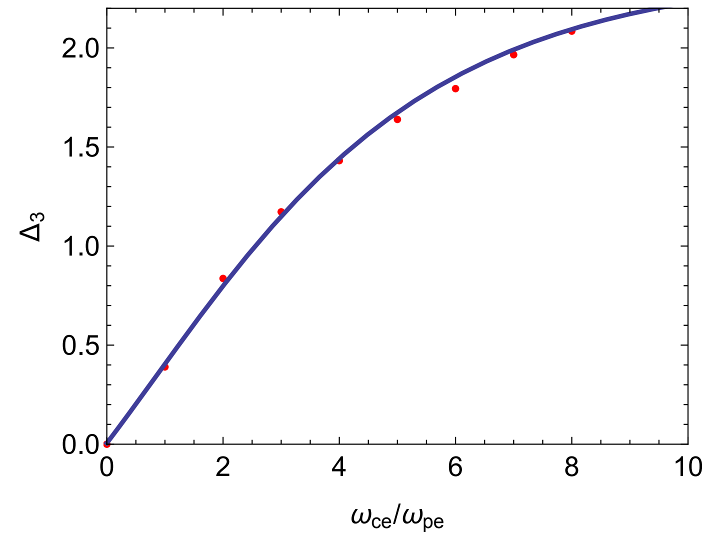
$$\text{Magnetic pumping: } F_{TTMP} = -\mu \nabla B_z$$

- Note that for electron moving near the speed of light, the effect from the electric force (E) and Lorentz force ($v \times B/c$) can be of similar order.

TTMP momentum loss

- From Faraday's law, $B_z = k_x cE_y/\omega$. So the TTMP force can be calculated similarly from E_y.

$$F_{TTMP} = \frac{\mu\omega_{ce}}{c^2\omega_{pe}}\frac{\omega_{pe}^2}{v^2}\left[\ln\left(\frac{v}{v_{th}}\right) - \frac{1}{2}\left(1 - \frac{v_{th}}{v}\right) + \Delta_3\right]$$



- The ratio of TTMP over polarization electric force

$$\frac{F_{TTMP}}{eE^p} \approx \gamma\left(\frac{v_\perp}{c}\right)^2$$

- For high energy regime where synchrotron radiation energy loss dominates, $v_\perp/c \sim 1/\gamma$, the effect of TTMP is small.
  - For high energy runaway electrons, synchrotron and Bremsstrahlung radiation dominate
- With fast pitch angle scattering mechanism (different from collisional scattering) such as whistler wave, the effect can be more important.

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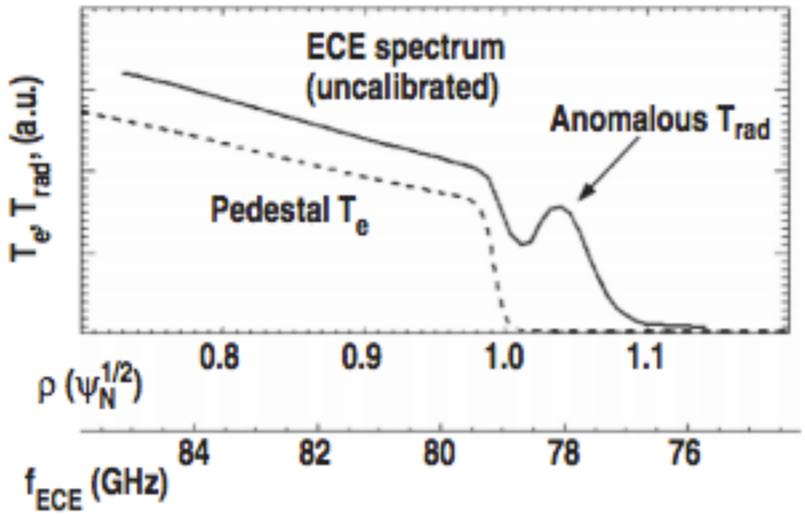
- Cherenkov radiation causes runaway electrons to lose energy, which can be described by adding a correction to the Coulomb logarithm in the drag force.
  - The correction is about 20% compared to collisional force in DIII-D QRE experiment
- Magnetic field enhances Cherenkov radiation and energy loss.
  - For runaway electrons with finite $v_\perp$, TTMP will cause momentum loss on parallel direction.
- Future work:
  - Including thermal effect in the plasma wave description
  - Study the spiral orbit particle rather than straight orbit
  - Study the electric field fluctuation given by Cherenkov radiation

Questions for discussion

1. Can we detect the Cherenkov radiation from runaway electrons?



B. J. Tobias et al., Rev. Sci. Instrum. 83, 10E329 (2012)

2. What is the correct physics interpretation of Cherenkov resonance condition when collisional damping is considered?

$$\omega = k \cdot v \qquad \exp(ikx - i\omega t) \quad \begin{cases} \operatorname{Im}\omega < 0, \operatorname{Im}k < 0 & \text{Wave is damping in time, but growing in space?} \\ \operatorname{Im}\omega > 0, \operatorname{Im}k > 0 & \text{Wave is damping in space, but growing in time?} \end{cases}$$