(*)

$$\varepsilon_{ijk}\cdot\varepsilon_{lmn} = \begin{vmatrix} a_{1i} & a_{1j} & a_{1k} \\ a_{2i} & a_{2j} & a_{2k} \\ a_{3i} & a_{3j} & a_{3k} \end{vmatrix} \cdot \begin{vmatrix} a_{1l} & a_{1m} & a_{1n} \\ a_{2l} & a_{2m} & a_{2n} \\ a_{3l} & a_{3m} & a_{3n} \end{vmatrix} =$$

$$= \begin{vmatrix} a_{1i} & a_{2i} & a_{3i} \\ a_{1j} & a_{2j} & a_{3j} \\ a_{1k} & a_{2k} & a_{3k} \end{vmatrix} \cdot \begin{vmatrix} a_{1l} & a_{1m} & a_{1n} \\ a_{2l} & a_{2m} & a_{2n} \\ a_{3l} & a_{3m} & a_{3n} \end{vmatrix} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix} =$$

$$= \delta_{il}\delta_{jm}\delta_{kn} - \delta_{il}\delta_{km}\delta_{jn} - \delta_{im}\delta_{jl}\delta_{kn} + \delta_{im}\delta_{kl}\delta_{jn}$$
$$+ \delta_{in}\delta_{jl}\delta_{km} - \delta_{in}\delta_{kl}\delta_{jm}$$

Zúžení v indexech k, n

$$\varepsilon_{ijk}\cdot\varepsilon_{lmk} = \varepsilon_{kij}\cdot\varepsilon_{klm} = 3\delta_{il}\delta_{jm} - \delta_{il}\cdot\delta_{km}\delta_{jk} - 3\delta_{im}\delta_{jl}$$
$$+ \delta_{im}\delta_{kl}\cdot\delta_{jk} + \delta_{ik}\cdot\delta_{jl}\cdot\delta_{km} - \delta_{ik}\delta_{kl}\,\delta_{jm} =$$
$$= \underline{3\delta_{il}\cdot\delta_{jm}} - \underline{\delta_{il}\cdot\delta_{jm}} - 3\,\delta_{im}\cdot\delta_{jl} + \delta_{im}\cdot\delta_{jl} + \delta_{im}\cdot\delta_{jl} -$$
$$- \underline{\delta_{il}\cdot\delta_{jm}} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

(*) Pro Levi-Civitův symbol platí

$$\varepsilon_{ijk} = \begin{vmatrix} a_{1i} & a_{1j} & a_{1k} \\ a_{2i} & a_{2j} & a_{2k} \\ a_{3i} & a_{3j} & a_{3k} \end{vmatrix}$$

kde a_{ij} jsou koeficienty ortogonální transformace $x'_i = a_{ij}x_j$

vysvětlení:
Pokud jsou některé 2 indexy stejné, determinant je nulový, pokud provedeme permutaci řádků, jeho znaménko se změní v souladu s definicí ε_{ijk}.