Value of the Cosmological Constant: Theory versus Experiment

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Abstract. The numerical value of the cosmological constant is calculated using a recently suggested cosmological model and found to be $\Lambda = 2.036 \times 10^{-35}\text{s}^{-2}$. This value of Λ is in excellent agreement with the measurements recently obtained by the *High-Z Supernova Team* and the *Supernova Cosmology Project*.

The problem of the cosmological constant and the vacuum energy associated with it is of high interest these days. There are many questions related to it at the quantum level, all of which are related to quantum gravity. Why there exists the critical mass density and why the cosmological constant has this value? Trying to answer these questions and others were recently the subject of many publications [1–13].

In this paper it is shown that the recently suggested cosmological model [14] predicts the value $\Lambda = 2.036 \times 10^{-35}\text{s}^{-2}$ for the cosmological constant. This value of Λ is in excellent agreement with the measurements recently obtained by the *High-Z Supernova Team* and the *Supernova Cosmological Project* [15–21].

The Einstein gravitational field equations with the added cosmological term are [22]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{1}$$

where Λ is the cosmological constant, the value of which is supposed to be determined by experiment. In Eq. (1) $R_{\mu\nu}$ and R are the Ricci tensor and scalar, respectively, $\kappa = 8\pi G$, where G is Newton's constant and the speed of light is taken as unity.

Recently the two groups (the *Supernovae Cosmology Project* and the *High-Z Supernova Team*) concluded that the expansion of the universe is accelerating [15–21]. Both teams obtained

$$\Omega_M \approx 0.3, \quad \Omega_\Lambda \approx 0.7, \tag{2}$$

and ruled out the traditional (Ω_M, Ω_Λ)=(1, 0) universe. Their value of the density parameter Ω_Λ corresponds to a cosmological constant that is small but, nevertheless, nonzero and positive,

$$\Lambda \approx 10^{-52}\mathrm{m}^{-2} \approx 10^{-35}\mathrm{s}^{-2}. \tag{3}$$

In Ref. 14 a four-dimensional cosmological model was presented. The model predicts that the universe accelerates and hence it is equivalent to having a positive value for cosmological constant in it. In the framework of this model the zero-zero component of Einstein's equations is written as

$$R_0^0 - \frac{1}{2}\delta_0^0 R = \kappa\rho_{eff} = \kappa\left(\rho - \rho_c^{BC}\right) \tag{4}$$

where $\rho_c^{BC} = 3/\kappa\tau^2$ is the critical mass density and τ is Hubble's time in the zero-gravity limit.

Comparing Eq. (4) with the zero-zero component of Eq. (1), one obtains the expression for the cosmological constant,

$$\Lambda = \kappa\rho_c^{BC} = 3/\tau^2. \tag{5}$$

To find out the numerical value of τ we use the relationship between $h = \tau^{-1}$ and H_0 given in Ref. 14 [Eq. (5.23)]:

$$H_0 = h\left[1 - \left(1 - \Omega_M^{BC}\right)z^2/6\right], \tag{6}$$

where z is the redshift and $\Omega_M^{BC} = \rho_M/\rho_c^{BC}$ where $\rho_c^{BC} = 3h^2/8\pi G$ [14]. (Notice that ρ_c^{BC} is different from the standard ρ_c defined with H_0.) The redshift parameter z determines the distance at which H_0 is measured. We choose $z = 1$ (Fig. 11 in Ref. 14) and take for

$$\Omega_M^{BC} = 0.245 \tag{7}$$

(roughly corresponds to 0.3 in the standard theory), Eq. (6) then gives

$$H_0 = 0.874h. \tag{8}$$

At the value $z = 1$ the corresponding Hubble constant H_0 according to the latest results from HST can be taken [23] as $H_0 = 72$km/s-Mpc, thus $h =$ (72/0.874)km/s-Mpc or

$$h = 82.380\text{km/s-Mpc}, \tag{9}$$

and

$$\tau = 12.16 \times 10^9\text{years}. \tag{10}$$

What is left is to find the value of Ω_Λ^{BC}. We have $\Omega_\Lambda^{BC} = \rho_c^{ST}/\rho_c^{BC}$, where $\rho_c^{ST} = 3H_0^2/8\pi G$ and $\rho_c^{BC} = 3h^2/8\pi G$. Thus $\Omega_\Lambda^{BC} = (H_0/h)^2 = 0.874^2$, or

$$\Omega_\Lambda^{BC} = 0.764. \tag{11}$$

As is seen from Eqs. (7) and (11) one has

$$\Omega_M^{BC} + \Omega_\Lambda^{BC} = 1.009 \approx 1, \tag{12}$$

which means the universe is flat.

As a final result we calculate the cosmological constant according to Eq. (5). One obtains

$$\Lambda = 3/\tau^2 = 2.036 \times 10^{-35} s^{-2}. \tag{13}$$

Our results confirm those of the supernovae experiments and indicate on existance of the dark energy as has recently received confirmation from the Boomerang cosmic microwave background experiment [24,25], which showed that the universe is flat.

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