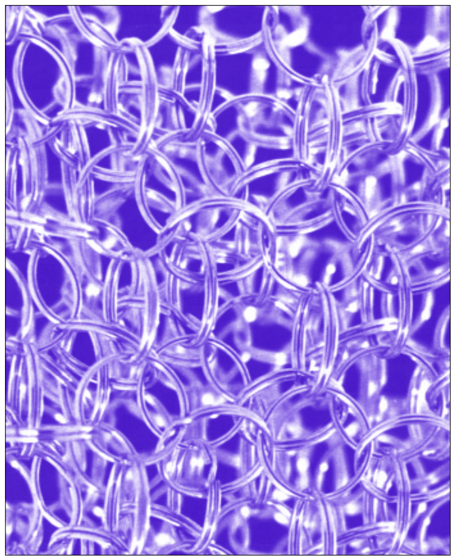
Loop gravity combines general relativity and quantum theory but it leaves no room for space as we know it – only networks of loops that turn space–time into spinfoam

Loop quantum gravity

Carlo Rovelli

GENERAL relativity and quantum theory have profoundly changed our view of the world. Furthermore, both theories have been verified to extraordinary accuracy in the last several decades. Loop quantum gravity takes this novel view of the world seriously, by incorporating the notions of space and time from general relativity directly into quantum field theory. The theory that results is radically different from conventional quantum field theory. Not only does it provide a precise mathematical picture of quantum space and time, but it also offers a solution to long-standing problems such as the thermodynamics of black holes and the physics of the Big Bang.



Weaving space – the 3D structure of space in loop quantum gravity can be visualized as a net of intersecting loops. This simple model was built by the author using key-rings, before spin networks and the physical significance of the nodes were discovered.

The most appealing aspect of loop quantum gravity is that it predicts that space is not infinitely divisible, but that it has a granular structure. The size of these elementary "quanta of space" can be computed explicitly within the theory, in an analogous way to the energy levels of the hydrogen atom. In the last 50 years or so, many approaches to constructing a quantum theory of gravity have been explored, but only two have reached a full mathematical description of the quantum properties of the gravitational field: loop gravity and string theory. The last decade has seen major advances in both loop gravity and string theory, but it is important to stress that both theories harbour unresolved issues. More importantly, neither of them has been tested experimentally. There is hope that direct experimental support might come soon, but for the moment either theory could be right, partially right or simply wrong. However, the fact that we have two well developed, tentative theories of quantum gravity is very encouraging. We are not completely in the dark, nor lost in a multitude of alternative theories, and quantum gravity offers a fascinating glimpse of the fundamental structure of nature.

Space and quantum space

Loop quantum gravity changes the way we think about the structure of space. To illustrate this, let me start by recalling some basic ideas about the notion of space and the way these were modified by general relativity. Space is commonly thought of as a fixed background that has a geometrical structure – as a sort of "stage" on which matter moves independently. This way of understanding space is not, however, as old as you might think; it was introduced by Isaac Newton in the 17th century. Indeed, the dominant view of space that was held from the time of Aristotle to that of Descartes was that there is no space without matter. Space was an abstraction of the fact that some parts of matter can be in touch with others.

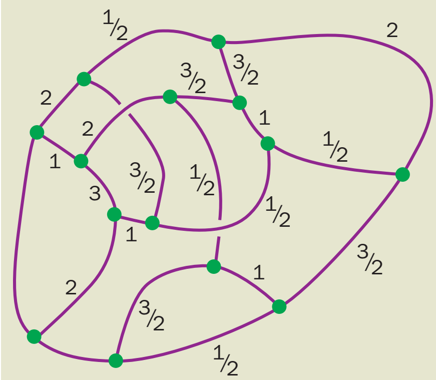
Newton introduced the idea of physical space as an independent entity because he needed it for his dynamical theory. In order for his second law of motion to make any sense, acceleration must make sense. Newton assumed that there is a physical background space with respect to which acceleration is defined. The Newtonian picture of the world is therefore a background space on which matter moves.

A small but momentous change in the Newtonian picture came from the visionary work of Michael Faraday and James Clerk Maxwell at the end of the 19th century. Faraday and Maxwell introduced a novel object that could move in space. This object was called the field, and Faraday visualized it as a set of lines that fill space. The lines start and end on electric charges, but they can exist and have independent dynamics even when no charges are present. In this latter case the field lines have no ends, and therefore form closed loops. Maxwell then translated Faraday's intuition into equations, in which these lines and loops became the electric and magnetic fields.

A few decades later Albert Einstein came up with special relativity, in which the geometry of space and time is slightly modified to make it compatible with Maxwell's field equations. Today our basic understanding of the material world is entirely in terms of fields. The fundamental forces in nature are described by Yang–Mills fields, which are similar to the electromagnetic field. Fundamental particles, such as quarks and electrons, are described by "fermionic" fields, and Higgs particles, which endow particles with mass, are described by "scalar" fields. Quantum field theory tells us that all fields undergo quantum fluctuations and have particle-like properties. In the Standard Model of particle physics – which comprises the quantum field theories of electromagnetism and the strong and weak nuclear forces – these fields are assumed to exist against a fixed background space–time that is similar to that described by Newton.

The truly major change in our understanding of space and time came with general relativity. In 1915 Einstein realized that gravity also had to be described by a field theory in order to be consistent with special relativity. He succeeded in finding the form of the gravitational field and its field equations, but in doing so he stumbled upon an extraordinary result. Einstein found that the gravitational field that he had just introduced and the background space that Newton had introduced 300 years earlier are, in fact, the same thing. The acceleration in Newton's second law is not with respect to an absolute background space, but with respect to the surrounding gravitational field. Newton had mistaken the surrounding gravitational field for a fixed entity. In general relativity there are no fields on space–time, just fields on fields.

1 Spin network



Elementary grains of space are represented by the nodes on a "spin network" (green dots). The lines joining the nodes, or adjacent grains of space, are called links. Spins on the links (integer or half-integer numbers) are the quantum numbers that determine the area of the elementary surfaces separating adjacent grains of space. The quantum numbers of the nodes, which determine the volume of the grains, are not indicated. The spins and the way they come together at the nodes can take on any integer or half-integer value, and are governed by the same algebra as angular momentum in quantum mechanics.

J BAEZ

As long as we stay within the classical regime, rather than the quantum one, the gravitational field defines a 4D continuum. We can therefore still think of the field as a sort of space–time, albeit one that bends, oscillates and obeys field equations. However, once we bring quantum mechanics into the picture this continuum breaks down. Quantum fields have a granular structure – the electromagnetic field, for example, consists of photons – and they undergo probabilistic fluctuations. It is difficult to think of space as a granular and fluctuating object. We can, of course, still call it "space", or "quantum space", as indeed I do in this article. But it is really a quantum field in a world where there are only fields over fields, and no remnant of background space.

Loops on loops

The conventional mathematical formalism of quantum field theory relies very much on the existence of background space. There are therefore two possible strategies that we can adopt to construct a quantum theory of gravity. One is to undo Einstein's discovery and to reintroduce a fictitious background space. This can be done by separating the gravitational field into the sum of two components: one component is regarded as a background, while the other is treated as the quantum field. We are then left with a background space that is available for all our calculations, after which we can hope to recover background independence. This is the strategy adopted by those who do not regard the general-relativistic revolution as fundamental, but as a sort of accident. And this is the strategy adopted in string theory.

The second strategy is the one adopted by loop gravity: take general relativity seriously, directly face the problem that there is no background space in nature, and reconstruct quantum field theory from scratch in a form that does not require background space. General ideas on how to do this were put forward in the 1950s and 1960s. Charles Misner, now at the University of Maryland, for example, suggested using Feynman's version of quantum field theory, in which the behaviour of a quantum particle can be calculated by summing all the possible classical paths of the particle. Misner suggested that calculations in quantum gravity could be performed by summing over all possible space–times – an idea that was later developed by theorists that included Steven Hawking at Cambridge University and Jim Hartle at the University of California in Santa Barbara.

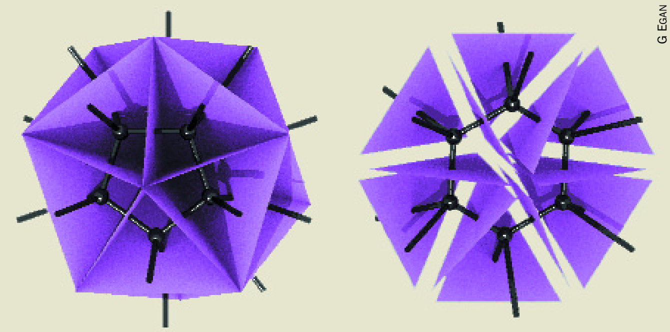
John Wheeler of Princeton University suggested that space–time must have a foam-like structure at very small scales and, along with Bryce DeWitt now at Texas University, he introduced the idea of a "wavefunction over geometries". This is a function that expresses the probability of having one space–time geometry rather than another, in the same way that the Schrödinger wavefunction expresses the probability that a quantum particle is either here or there. This wavefunction over geometries obeys a very complicated equation that is now called the Wheeler–DeWitt equation, which is a sort of Schrödinger equation for the gravitational field itself. It is important, however, not to confuse the dynamics *in* a gravitational field with the dynamics *of* the gravitational field itself. (The difference between the two is the same as the difference between the equation of motion for a particle in an electromagnetic field and the Maxwell equations for the electromagnetic field itself.)

These ideas were brilliant and inspiring, but it was more than two decades before they become concrete. The turnaround came suddenly at the end of the 1980s, when a well defined mathematical theory that described quantum space–time began to form. The key input that made the theory work was an old idea from particle physics: the natural variables for describing a Yang–Mills field theory are precisely Faraday's "lines of force". A Faraday line can be viewed as an elementary quantum excitation of the field, and in the absence of charges these lines must close on themselves to form loops. Loop quantum gravity is the mathematical description of the quantum gravitational field in terms of these loops. That is, the loops are quantum excitations of the Faraday lines of force of the gravitational field. In low-energy approximations of the theory, these loops appear as gravitons – the fundamental particles that carry the gravitational force. This is much the same way that phonons appear in solid-state physics. In other words, gravitons are not in the fundamental theory – as one might expect when trying to formulate a theory of quantum gravity – but they describe collective behaviour at large scales.

The idea that loops are the most natural variables to describe Yang–Mills fields has attracted the attention of many theoretical physicists, including Kenneth Wilson at Ohio State University, Alexander Polyakov at Princeton, Stanley Mandelstam at Berkeley and Rodolfo Gambini at the University of Montevideo. But in the past the idea has never really worked well. Two loops that are infinitesimally separated are two different loops, and this implies that there are far too many loop variables to describe the degrees of freedom of the field.

The breakthrough came with the realization that this "overcounting" problem disappears in gravity. The reason why is not hard to understand. In gravity the loops themselves are not in space because there is no space. The loops *are* space because they are the quantum excitations of the gravitational field, which is the physical space. It therefore makes no sense to think of a loop being displaced by a small amount in space. There is only sense in the relative location of a loop with respect to other loops, and the location of a loop with respect to the surrounding space is only determined by the other loops it intersects. A state of space is therefore described by a net of intersecting loops. There is no location *of* the net, but only location *on* the net itself; there are no loops on space, only loops on loops. Loops interact with particles in the same way as, say, a photon interacts with an electron, except that the two are not in space like photons and electrons are. This is similar to the interaction of a particle with Newton's background space, which "guides" it in a straight line.

2 Quantum loops



Each node in a spin network determines a cell, or an elementary grain of space. (*a*) Nodes are represented by small black spheres and the links as black lines, while cells are separated by elementary surfaces shown in purple. Each surface corresponds to one link, and the structure builds up a 3D space. (*b*) When the surfaces are pulled away we can see that the sequence of links form a loop. These are the "loops" of loop quantum gravity.

Spin networks

In 1987 I visited Lee Smolin at Yale University. Smolin and Ted Jacobson of the University of Maryland had been working on an approximation to quantum gravity, and had found some solutions of the Wheeler–DeWitt equation that seemed to describe loop excitations of the gravitational field. Smolin and I decided to write down the entire theory systematically in loop variables, and we were shocked by a remarkable series of surprises. First, the formerly intractable Wheeler–DeWitt equation became tractable, and we could find a large class of exact solutions. Second, we had a workable formalism for a truly background-independent quantum field theory.

We used a novel formulation of general relativity that was due to Abhay Ashtekar of Penn State University, who had cast general relativity in a very similar form to Yang–Mills theory. Einstein's gravitational field is replaced by a field called the Ashtekar connection field, which is like the electromagnetic potential, and this made loop variables very natural. Smolin and I teamed up with Ashtekar to try and understand the physical meaning of the nets of loops that had emerged from the equations. Through various steps we slowly realized that the loops did not describe infinitesimal elements of space as we had first thought, but rather finite elements of space. We pictured space as a sort of extremely fine fabric that was "weaved" by the loops. Nothing appeared to exist at scales smaller than the structure of the weave itself.

The idea that there cannot be arbitrary small spatial regions can be understood from simple considerations of quantum mechanics and classical general relativity. The uncertainty principle states that in order to observe a small region of space–time we need to concentrate a large amount of energy and momentum. However, general relativity implies that if we concentrate too much energy and momentum in a small region, that region will collapse into a black hole and disappear. Putting in the numbers, we find that the minimum size of such a region is of the order of the Planck length – about 1.6×10^{-35} m. Loop gravity had begun to make this intuition concrete, and a picture of quantum space in terms of nets of loops was emerging. But at the time we did not really understand what that meant. Jorge Pullin of Louisiana State University, for instance, remarked that we were not really understanding the volume of space, and instead pointed to the "nodes" – the points at which loops intersect – as the structure that had to be connected with the volume.

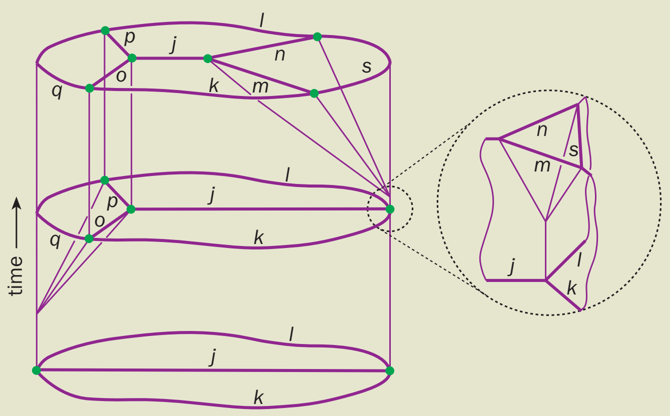
It was not until about 1994 that Smolin and I really understood what we had stumbled upon, thanks to a calculation that is routinely performed in quantum theory. By quantizing a theory, certain physical quantities take only discrete values, such as the energy levels in the hydrogen atom. Computing these quantized values involves solving the eigenvalue problem for the "operator" that represents a particular physical quantity. We studied the volume of a region of space – or a certain number of loops – which in general relativity is determined by the gravitational field. By solving the eigenvalue problem of the volume operator, we found that the eigenvalues were discrete – that is, there are elementary quanta of volume, or elementary "grains of space". Furthermore, these quanta of space resided precisely at the nodes of the nets.

But space is more than just a collection of volume elements. There is also the key fact that some elements are near to others. A "link" of the net – i.e. the portion of loop between two nodes – indicates precisely the quanta of space that are adjacent to one another. Two adjacent elements of space are separated by a surface, and the area of this surface turns out to be quantized as well. In fact, it soon became clear that nodes carry quantum numbers of volume elements and links carry quantum numbers of area elements (figure 1).

While unravelling this elegant mathematical description of quantum space, we realized that we had come across something that had already been studied. Some 15 years earlier, Roger Penrose of Oxford University – guided only by his intuition of what a quantum space could look like – had invented precisely the nets carrying the very same quantum numbers that we were finding. Since these quantum numbers and their algebra looked like the spin angular momentum numbers of elementary particles, Penrose called them "spin networks" (figure 2). Penrose had invented spin networks out of the blue, but we were finding the same networks from a direct application of quantum theory to general relativity. It was with Penrose's help during a summer in Verona, Italy, in 1994 that Smolin and I finally solved the problem of the eigenvalues of area and volume.

Meanwhile, Chris Isham of Imperial College in London, who was one of the founding fathers of the background-inde-

3 Spinfoam



Loop quantum gravity replaces the Newtonian concept of background space with a history of spin networks called a spinfoam. Each link in the network is associated with a quantum number of area called "spin", which is measured in units related to the Planck length. Here a θ-shaped spin network (bottom) with three links carrying spins *j*, *k* and *l* evolves in two steps into a spin network carrying spins *o*, *p*, *q*, *j*, *k*, *l*, *m*, *n* and *s* (top). The initial spin network has two nodes where the three links meet, and the vertical lines from these nodes define the edges of the spinfoam. The first vertex – which is similar to the vertex of a Feynman diagram – is where the left edge branches off, at which point an intermediate spin network with spins *o*, *p*, *q*, *j*, *k* and *l* is formed. The edge on the right branches off in a second interaction vertex, which is enlarged. The "faces" of the spinfoam are the surfaces swept by the links moving in time. The enlargement shows that the vertex is connected to four edges and six faces with associated spins *j*, *k*, *l*, *m*, *n* and *s*. Spinfoams like this one can be thought of as a discretized quantum space–time.

pendent approach to quantum gravity, along with Ashtekar and Jerzy Lewandowski of Warsaw University had begun to develop mathematically rigorous foundations of the theory. Together with several other physicists and mathematicians, they were able to re-derive and extend the results that we first found and give them a solid grounding. Today, a vibrant community of theorists is developing the many aspects of loop quantum gravity.

The spin-networks picture of space–time is mathematically precise and physically compelling: nodes of spin networks represent elementary grains of space, and their volume is given by a quantum number that is associated with the node in units of the elementary Planck volume, $V=(\hbar G/c^3)^{3/2}$, where $\hbar$ is Planck's constant divided by 2π, G is the gravitational constant and c is the speed of light. Two nodes are adjacent if there is a link between the two, in which case they are separated by an elementary surface the area of which is determined by the quantum number associated with that link. Link quantum numbers, j, are integers or half-integers and the area of the elementary surface is $A = 16\pi V^{2/3}\sqrt{[j(j+1)]}$, where V is the Planck volume.

A physical region of space is in a quantum superposition of such spin-network states, and the dynamics in the region are governed by a well defined Wheeler–DeWitt equation – the mathematically rigorous form of which has been established by Thomas Thiemann at the Perimeter Institute in Waterloo. Remarkably, this simple picture follows from a rather straightforward application of quantum techniques to general relativity.

Spinfoam

Loop quantum gravity has numerous applications and results. For example, indirect semi-classical arguments suggest that a black hole has a temperature and therefore an entropy. This entropy, S, is given by the famous Bekenstein–Hawking formula, $S=Ak_Bc^3/4\hbar G$, where A is the area of the black hole and k_B is Boltzmann's constant. A long-standing problem in quantum gravity was to understand the temperature of black holes from first principles, and this formula has now been derived using loop gravity, albeit once a free parameter called the Immirzi parameter has been fixed.

Martin Bojowald at the Albert Einstein Institute in Berlin has recently been able to apply loop gravity to describe the physics of the Big Bang singularity. In cosmology the volume of the expanding universe plays the role of the time parameter. Since volume is quantized in loop gravity, the evolution of the universe takes place in discrete time intervals. The idea that cosmological time consists of elementary steps changes the behaviour of the universe drastically at very small scale, and gets rid of the initial Big Bang singularity. Bojowald has also found that an inflationary expansion might have been driven by quantum-gravitational effects. These developments are exciting, but they are just a taste of the full cosmological implications of loop gravity.

The eigenvalues of volume and area are also solid quantitative predictions of the theory. This means that any volume and area that we could measure should correspond to a particular number in a spin network. A direct test of this would require us to measure volumes or areas, such as cross-sections, with Planck-scale precision. This is currently well beyond our experimental ability, but it is reassuring that the theory makes definite quantitative predictions.

The granular structure of space that is implied by spin networks also realizes an old dream in theoretical particle physics – getting rid of the infinities that plague quantum field theory. These infinities come from integrating Feynman diagrams, which govern the probabilities that certain interactions occur in quantum field theory, over arbitrary small regions of space–time. But in loop gravity there are no arbitrary small regions of space–time. This remains true even if we add all the fields that describe the other forces and particles in nature to loop quantum gravity. Certain divergences in quantum chromodynamics, for example, disappear if the theory is coupled to the quantum gravitational field.

The mathematical control of the theory has also led to a well defined version of Misner and Hawking's' "sum over all possible space–times", which I described earlier. Space–time is a temporal sequence of spaces, or a history of spaces. In loop gravity, space is replaced by a spin network and space–time is therefore described by a history of spin networks. This history of spin networks is called "spinfoam", and it has a simple geometrical structure. The history of a point is a line, and the history of a line is a surface. A spinfoam is therefore formed by surfaces called faces, which are the histories of the links of the spin network, and lines called edges, which are the histories of the nodes of the spin network (figure 3).

Faces meet at edges, which, in turn, meet at vertices. These vertices represent elementary interactions between the nodes – namely the interactions between the grains of space. Indeed, they are very similar to the vertices in Feynman diagrams, which represent interactions between particles in conventional quantum field theory. In loop gravity, space–time can be viewed as a Feynman diagram that represents the interactions of the grains of space. A spinfoam, however, is a bit more complicated than a Feynman diagram because it is formed by points, lines and surfaces, while a Feynman diagram has only points and lines.

In conventional quantum field theory, we sum over all possible Feynman diagrams, which are histories of interacting particles. In loop gravity, we sum over all spinfoams, which are histories of space–times, or histories of interacting grains of space. The term spinfoam was introduced by John Baez of the University of California at Riverside because it reminds us of Wheeler's idea that quantum space–time has a foam-like structure. A spinfoam is indeed a mathematically precise realization of Wheeler's intuition. In a particular spinfoam formulation that was initiated by Louis Crane of Kansas University and John Barrett of the University of Nottingham, key convergence theorems have been proven by Alejandro Perez of Penn Sate University. Today their model is extensively explored as a promising way to derive physical predictions from loop gravity.

J Ambjorn, Niels Bohr Institute, K N Anagnostopoulos, University of Crete and R Loll, Max Planck Institute for Gravitational Physics

Loop quantum gravity predicts that space–time is made up of elementary grains of volume at the Planck scale. Numerical simulations like this one can reveal how such discreet quantum geometries "evolve" into smooth classical space.

In recent years it has become increasingly clear that some quantum-gravity effects might be observable with existing experimental technology. Of these, the most promising is the possibility of detecting violations of Lorentz invariance at very high energy due to quantum-gravity effects at small scales. The granular structure of space would mean that different wavelengths of light could travel at different speeds – as they do in crystals – and therefore violate Lorentz invariance, which demands that all photons travel at the speed of light. Such a mechanism could play a role, for example, in the unexplained energy thresholds of cosmic rays (see article on page xx).

However, the violation of Lorentz invariance is only a possibility in loop gravity, not a strict prediction of the theory. The theory is therefore not in contradiction with recent observational limits on violations of Lorentz invariance from measurements of cosmic gamma rays by Floyd Stecker at NASA's Goddard Space Flight Center and Ted Jacobson at the University of Maryland. But observations such as these lay the old worry about testing a quantum theory of gravity to rest. Today quantum-gravity theorists, like all physicists, wait anxiously for new observational data.

Testing times

So, does this mean that all is well in loop quantum gravity? Not at all. Some aspects of the theory are still unclear. The key dynamical equation of the theory – the Wheeler–DeWitt equation – exists in several varieties and we do not know which, if any, is the correct one. The connection to low-energy physics is also unclear. What is missing is a systematic way of computing scattering amplitudes and cross-sections, such as the standard perturbation expansion in quantum field theory. The mathematics of the theory is well defined, but this does not mean we know how to calculate everything.

Furthermore, the theory contains an odd parameter called the Immirzi parameter, γ, which is not fixed. The freedom in choosing this parameter was emphasized by Giorgio Immirzi at the University of Perugia in Italy, and at present it is fixed indirectly by requiring the theory to agree with the Bekenstein–Hawking black-hole entropy. This is nontrivial, since the same value of γ matches many different kinds of black holes, and there is some indication that the same value could be obtained in other ways as well. But such an indirect way of determining the Immirzi parameter is not satisfactory, and there is something we do not yet understand in this respect.

Finally, I repeat that for the moment there has not been any direct experimental test of the theory. A theoretical construction must remain humble until its predictions have been directly and unambiguously tested. This is true for strings as well as for loops. Nature does not always share our tastes about a beautiful theory. Maxwell's theory became credible when radio waves were observed. General relativity became credible when the deflection of the light by the Sun was measured and when atomic clocks in the Global Positioning Satellite system were found to run faster than they do on Earth.

The Standard Model of particle physics became credible when the intermediate W and Z bosons were found, right where the theory predicted, and when innumerable cross-sections turned out to match experiment extraordinary well. Nothing of the sort has happened in post-Standard Model physics. The proton is not decaying in the way it was predicted. Supersymmetry has not been found where it was expected to be. The predicted effects of higher dimensions of space–time have not shown up.

The advantage of loop quantum gravity is that it does not need unobserved supersymmetry, proton decay, higher dimensions, or similar in order to provide a coherent picture of quantum space–time. The reason why I think that loop quantum gravity is the right way forward is that it provides a theoretical structure that fully incorporates the deep lessons of general relativity.

General relativity is not about physics on curved space–times, asymptotic space–times, or connections between theories defined over different backgrounds. It is the discovery that there is no background; no space–time. The challenge for the physicists of the 21st century is to complete the scientific revolution that was started by general relativity and quantum theory. For this we must understand quantum field theory in the absence of a background space–time. Loop quantum gravity is the most resolute attempt to address this problem.

Further reading

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