The Mechanism of Sliding on Ice and Snow

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The mechanism of sliding on ice and snow

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Introduction

The preceding paper has shown that the sliding friction between metals decreases in a regular manner as the temperature is raised, provided that the chemical nature of the surface layers remains unchanged. The behaviour of the friction at temperatures near the melting-point was not investigated. Most metals become very soft at these high temperatures, and the irregularities which normally separate the two surfaces now collapse, so that a comparatively large area of contact is developed. Since the frictional force is the resultant of the minute adhesions acting across the real area of contact, we should expect that the friction between two such surfaces would be very great. The friction between two gold surfaces sliding together has been observed to increase to very high values where the metal becomes soft.

With solids of different physical properties, however, there is the further possibility that a surface layer of solid may be melted, either by pressure melting or by frictional heating, when the mass of the solid is still frozen and hard. If the melted fluid is present in sufficient quantity and possesses a low viscosity, the conditions may become those of lubricated sliding and lead to a large reduction in the friction. This effect would naturally be most marked for those substances which retain their rigid character at temperatures close to the melting-point, and which give a liquid of low viscosity on melting. Ice is a solid which has these properties, and it is well known that the friction of solids on ice and snow is very low. The suggestion has often been made that, in skating or ski-ing, the surfaces are lubricated by a layer of water formed by pressure melting (e.g. Reynolds 1901, p. 737), but few measurements of any sort have been made either to support or to disprove this suggestion.

Pressure melting. Experience shows that ski slide quite readily on snow at $-20°$ C. If we consider that the ski makes contact with the snow over the whole of its under surface (neglecting the tip), the pressure for an average man of 75 kg. weight on ski of area 5000 cm.^{2} is 15 g./cm.^{2}. This

[280]

pressure can only form a water layer if the snow were at $-0.00012°$ C or a higher temperature. The real area of contact is of course less than the apparent area. In the case of metals the real area may be a minute fraction of the apparent area of contact, and the mutual pressure between the two surfaces may be the flow pressure of the metal (Holm 1929; Bowden and Tabor 1939). With a powder such as snow, which can pack down and conform to the shape of the ski, we might expect the area of contact to be greater—perhaps not less than 1/1000 of the area of the ski. If this were the case then the pressure is still only great enough to melt snow at $-0.12°$ C. In order to have a sufficient pressure to melt snow at the low temperature of $-20°$ C, the real area of contact must be less than 0.031 cm.[2], i.e. about 1/100,000 of its apparent area.

These calculations of the pressure melting are made on the assumption that the pressure is applied equally to the solid and to the liquid phases. Poynting (1881) has pointed out that if the pressure is applied to the solid alone, the lowering of the melting-point is about $11\frac{1}{2}$ times as great. This effect and its bearing on flow pressures has been discussed by Johnston (1912) and Jeffreys (1935). We cannot, on theoretical grounds rule out the pressure-melting theory, but if it is true it means that the real area of contact is extremely small. The pressure itself is, of course, not sufficient to cause melting. Heat must be supplied from some source which is at a higher temperature than that corresponding to the equilibrium melting pressure.

Melting due to frictional heating. There is, however, another possible cause of surface melting, that is the frictional heat liberated at the sliding interfaces. Earlier work (Bowden and Ridler 1936; Bowden and Hughes 1937) has shown that this frictional heat may raise the surface temperature of sliding metals to a high value and may cause local melting and flow of the solids. In the case of a ski sliding on ice at $-20°$ C, for example, if the coefficient of friction is $\mu = 0.05$ the amount of frictional heat liberated when the ski moves forward a distance of 1 cm. is given by

$$Q = \frac{\mu M g}{J} = \frac{0.05 \times 37.5 \times 981 \times 1000}{4.18 \times 10^7} = 0.044 \text{ cal.}$$

This heat is concentrated at the points of contact of the snow crystals. If the ski were making intermittent rubbing contact over its whole area, this heat would be sufficient to raise from -20 to $0°$ C and to melt a layer over 6 mol. in thickness over this area. In actual fact, the real area of contact must be very much less, and the heat is concentrated at the points of contact of the snow crystals. Again we do not know the real area of contact, but if the area of contact is taken as 0.1 cm.[2], then the thickness

of the water layer melted would be $5 \cdot 5 \times 10^{-3}$ cm. or $1 \cdot 7 \times 10^5$ molecular layers. It is clear that a great part of the heat must be lost by conduction from the points of contact to the surrounding snow and to the ski. These crude calculations cannot be taken very seriously but they do show that the frictional heat liberated is considerable, and that the retention of a small fraction of it may be sufficient to cause local melting at the points of contact of the snow or ice crystals.

This paper describes an investigation of the experimental laws which govern the friction of solids sliding on ice and snow. The work shows that these laws bear a general resemblance to those observed for other solids, but there are important differences. Experiments were undertaken to determine whether a water layer is formed at all, and if so, whether it is due to pressure melting or to frictional heating. The results show that at very low temperatures $(-140^\circ$ C) the friction of ice is high, and is of the same order as that observed for other unlubricated solids. At higher temperatures, however, the friction decreases and there is evidence that partial lubrication by a water film may occur. The results suggest, that, although pressure melting plays some part, the low friction observed for rapidly moving solids on ice and snow is due in a large measure to the melting of a water layer by frictional heating.

EXPERIMENTAL

The experiments were carried out in an ice cave dug out of the ice above the Research Station at the Jungfraujoch, 3346 m. in Switzerland. The temperature of the cave never rose above -3° C. Since the air is very pure at these altitudes, the danger of contamination of the surfaces by grease and vapour was very slight.

The apparatus used to study the friction is shown in fig. 1. It is similar in principle to that previously described by Bowden and Ridler (1936). A horizontal turntable (C) of copper carried the lower surface of ice or snow. This turntable was hollow, and could be filled with cooling agents such as solid carbon dioxide or liquid air, so that the film of ice on top of it could be cooled to any desired temperature. The upper sliding surface (I) was attached to the arm (A). The other end of this arm was held in gimbals (G) so that the sliding surface could move freely in a horizontal or a vertical plane. The load was applied by fitting weights in the cup (L). When the lower surface rotated the frictional force between the two surfaces would tend to drag the upper surface with it. This movement was prevented by a fine thread fastened to the upper slider at Z, and attached at the other

end to a flat spring. The deflection of this spring gave a measure of the frictional force between the two surfaces. It was measured by a beam of light reflected from a mirror attached to the spring.
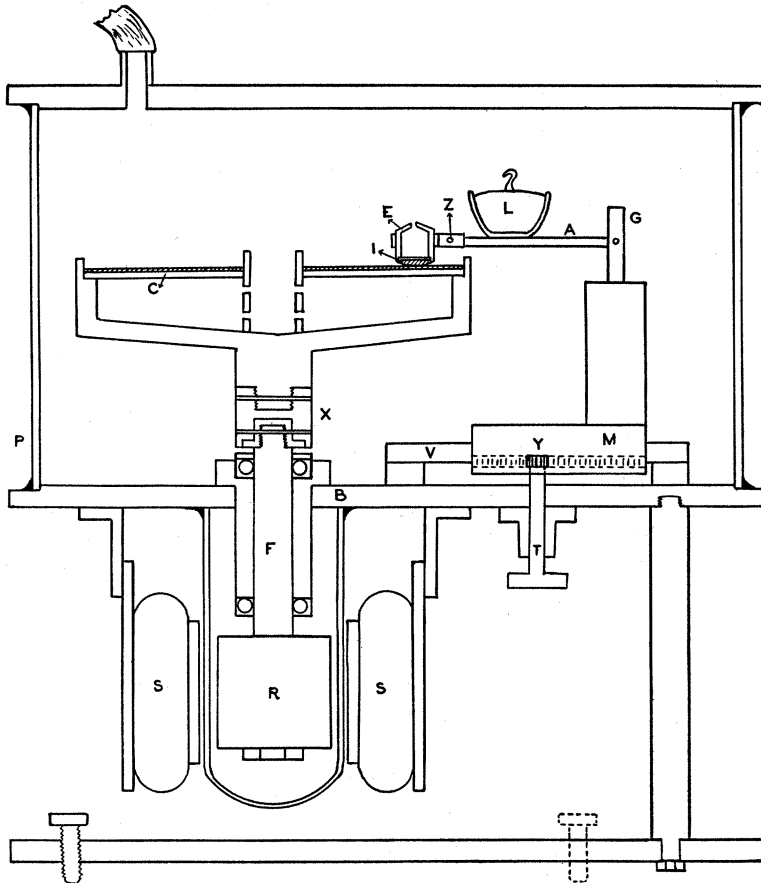

Fig. 1. The friction apparatus

The gimbals were mounted on the brass block (M) which could slide along the brass rail (V). This block was fitted underneath with a rack strip, and could be propelled along the rail by means of a small pinion (Y) on the tap (T). This sliding block also carried the vertical flat spring used for measuring the friction, and this was so arranged that its scale reading was not affected during the sliding of the block. When the tap (T) was turned, the upper slider (I) moved nearer to the centre of the turntable, so enabling the surfaces to run in a fresh track.

The apparatus was so designed that the sliding surfaces were completely enclosed in the glass cylinder (P) which could then be evacuated. The turntable (C) was connected to the rotor (R) of an induction motor by the vertical steel shaft (F). These two metal components were thermally insulated from one another by the block of vulcanite (X) secured by tapered pins. The rotor was separated from the stator (S) of the induction motor by a glass sheath sealed on to the flat steel base plate (B). This arrangement enabled the lower surface to be rotated in a vacuum. A number of experiments on the friction of ice surfaces were carried out in the absence and in the presence of air. No difference in the behaviour was observed, showing that the air of the laboratory caused no appreciable contamination of the surfaces.

<div align="center">RESULTS</div>

<div align="center">*The friction of ice on ice*</div>

The lower surface of ice was formed by running a film of water on to the turntable and allowing it to freeze. The table was then set in rotation and the ice surface was planed smooth with a small plane. The small upper ice surface (I) was frozen on to a copper plate which formed the base of an ebonite cylinder (E).

The influence of load on the friction.

The sliding speed was maintained constant at *ca.* 400 cm./sec. and the effect of load on the friction was investigated. Fig. 2, curve a, shows the results obtained with ice cooled to $-3°$ C. Over a certain range of loads (0–200 g.) there was a linear relation between the frictional force F and the applied load W showing that the coefficient of kinetic friction μ_k was constant. In this case μ_k was equal to 0·032. At higher loads the curve became concave to the W axis showing that μ_k decreased with higher loads. This type of curve was always obtained with ice surfaces cooler than $-1°$ C. Curve b shows a typical result for ice at $-27°$ C. The general relation is the same but the friction is higher, $\mu_k = 0·064$. If the ice was at $0°$ C, however, and the surfaces were allowed to run in the same track for some time, a different behaviour was observed (curve c). Under these circumstances the coefficient of friction μ_k increased at heavy loads and with the time of running. An examination of the surfaces showed that a thick film of water had formed on them. If this thick film of water was wiped off the lower surface with cotton-wool or was frozen with a block of carbon dioxide, the friction immediately fell again. If water was then

added to the surface the friction again rose. The addition of a small quantity of water to ice at $0°$ C, for example, raised the friction from $\mu_k = 0·024$ to $\mu_k = 0·075$. When this water was removed by freezing, the friction immediately fell to its initial value. This shows that the presence of a *thick* layer of water increases the friction between the rapidly moving surfaces. This thick layer can be formed by prolonged running on ice which is very nearly at $0°$ C. In order to prevent disturbances due to this effect measurements were made with short runs.
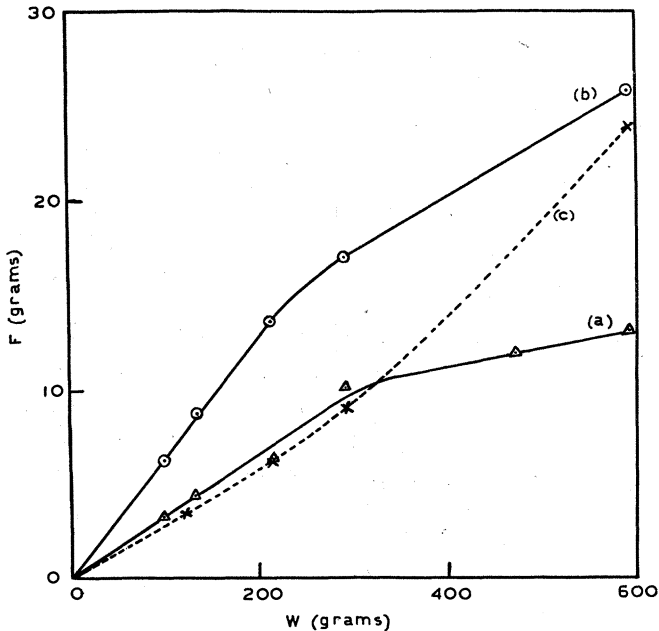

FIG. 2. The relation between the frictional force (F) and the load (W) for ice sliding on ice. Curve (*a*) mean surface temperature $-3·3°$ C; curve (*b*) mean surface temperature $-27·5°$; curve (*c*) mean surface temperature $0°$; long runs with the surfaces wet.

The influence of the area of contact on the friction.

The apparent area of contact was varied by changing the size of the top surface. It was not easy to carry out the experiments under identical conditions, but the results of a number of experiments are shown in Table I. The sliding speed in each case was 400 cm./sec., and the range of loads was from 0 to 300 g. The apparent area of contact varied from 0·02 to 3·1 cm.^{2}.

Owing to the difficulty of carrying out these experiments under exactly comparable conditions, the results must necessarily be only approximate. It will be seen, however, that although the apparent area of contact was

altered by a factor of 150, the coefficient of friction showed little change; this has also been observed with other solids (Beare and Bowden 1935).

If the ice was near 0° C and the upper surface was curved, prolonged running again caused an obvious melting or wearing away, so that a flat was worn on it and the area was increased to a large value. This had little effect on μ_k provided that a thick film of water was not allowed to form on the surface. If this thick film did form the friction again rose to a high value.

TABLE I

Exp. no.	Apparent area of contact cm.^{2}	Mean temperature ° C	μ_k
1	0·6	$-1\cdot4$	0·019
	2·3	$-2\cdot0$	0·019
2	0·6	$-3\cdot0$	0·017
	2·5	$-3\cdot0$	0·019
3	0·02	-1 to -10	0·016
	3·1	$-3\cdot0$	0·021

The influence of temperature on the friction.

If the low friction on ice is due to the formation of a thin water film, we should expect the friction to be a function of the temperature. The film would be more difficult to form at low temperatures and the friction should be higher. This is true whether its mechanism of formation is by pressure melting or by frictional heating. Experiments were carried out to investigate this point.

Both the ice surfaces could be cooled with liquid air or with ether and carbon dioxide. In practice, the surfaces were cooled to a low temperature and the friction measured at different intervals of time as the surfaces warmed up. The surface temperature of the ice was measured by pressing a special thermojunction of small heat capacity against the stationary surfaces, and the temperatures recorded are the mean of the temperatures taken before and after the friction measurements. The results for different loads are shown in fig. 3. The sliding speed in each case was 400 cm./sec. The friction at each temperature was measured with a variety of different surfaces and areas of contact. The values showed some variation and each curve is the average of a large number of determinations. It will be seen that in each case the kinetic friction increases as the temperature falls. Fig. 3, for a load of 130 g., for example, shows that the friction at $-140°$ C may be some five or six times greater than it is at 0° C. The value of the coefficient of friction ($\mu_k = 0\cdot11$) at these low temperatures is of the same order of magnitude as that observed on other crystalline solids. Hutchison

(1938 unpublished) has shown that the friction on calcite is $\mu_k = ca.\ 0.2$. In all three curves the effect of temperature on friction is much greater from 0 to $-40°$ C than over the range -40 to $-140°$ C. The effect of temperature is much greater on ice than it is on metals. Over the first part of the curve the average decrease in the friction of ice for a $10°$ rise in temperature was about 30 %. In the case of metals the decrease in the friction for a $10°$ rise was only about 0.05 %.
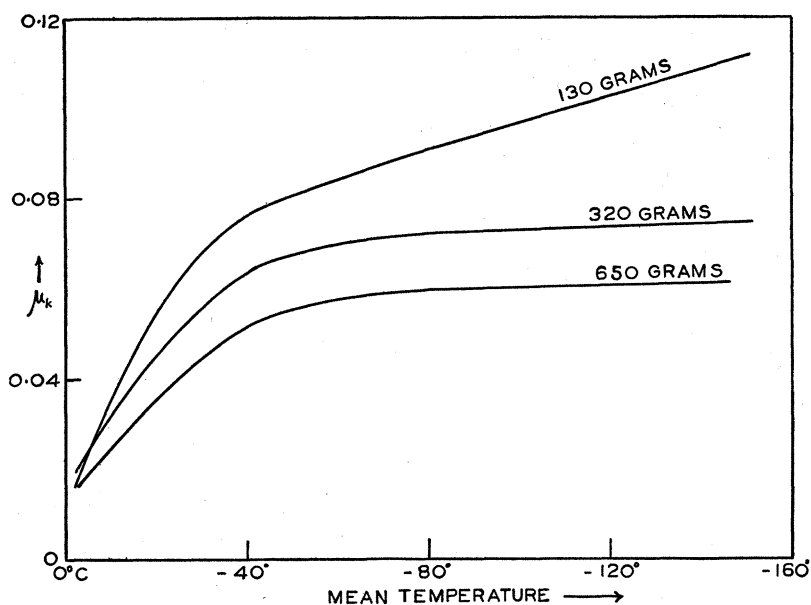

FIG. 3. The influence of temperature on the kinetic friction between ice surfaces.

Static and kinetic friction and the influence of speed.

The value of μ_s, the static friction, depended on the time for which the surfaces were allowed to remain in contact. This was particularly true for temperatures close to the melting-point. If the surfaces were allowed to stand in contact for some minutes at temperatures near $0°$ C, they became frozen together, and the force required to start motion was very great indeed. At temperatures well below the melting-point, this effect was not so marked. Although the values of the static friction were not very reproducible and showed some variation at different points, it was possible to obtain some idea of its general behaviour. The relation between the load and the frictional force was similar to that observed for kinetic friction (fig. 2). The F/W curve was at first linear showing that μ_s was independent

of the load, but at heavier loads the curve became concave to the load axis, showing that μ_s becomes less at heavier loads.

The effect of temperature on μ_s is shown in Table II.

TABLE II

Temperature, ° C ...	0	− 12	− 71	− 82	− 110
μ_s	0·05–0·15	0·3	0·5	0·5	0·5

Provided the ice was not actually melting the static friction was always much higher than the kinetic friction, and was of the same order of magnitude as that observed on other solids. At − 12° C, μ_s has the value 0·3 as compared with $\mu_k = 0·03$ under the same conditions. The results show that the static friction decreases as the temperature approaches the melting-point. At, or very close to 0° C, the results were very variable. If the ice was actually melting, so that a film of water was present, the force required to start the motion was very small indeed ($\mu_s = 0·02$). If, however, local freezing could occur, the static friction became too great for the apparatus to measure.

When the surfaces were set in motion the friction fell to the characteristic kinetic value, and this value was not greatly influenced by the speed of sliding until the speed became low. At very low speeds the coefficient of friction increased. An analysis of the sliding friction showed that, even at high speeds, μ_k was not constant but was fluctuating rapidly. It was clear that the surfaces were not sliding continuously, but were moving by a process of "stick and slip", and the general behaviour was very similar to that observed for metal surfaces (Bowden and Leben 1939). At temperatures near 0° C and at low speeds these fluctuations became very great indeed, and it was evident that an intermittent freezing together and breaking away was occurring.

If the pressure melting view is correct, there is no particular reason why the static friction should be so much higher than the kinetic friction. A water film should be present at the points of contact even though the surfaces are stationary. If, however, the water is melted by frictional heating we should expect the static friction to be very much higher than the kinetic friction. The results support the frictional heating mechanism. At first sight it might be thought that the kinetic friction should be very dependent upon velocity, and should decrease as the velocity increases since the rate of heat evolution and of melting should be greater. This is true for the slider but is not true for the lower surface. The slider is continuously moving forward on to a surface of fresh ice and the amount of

heat evolved per square centimetre of this surface is independent of the speed of sliding.

<p style="text-align: center;">*The friction of other solids on ice. The influence of*
thermal conductivity</p>

Experiments were carried out to study the friction of other solid surfaces sliding on ice. It was of particular interest to determine the influence that the thermal conductivity of the ski would have on the friction at low temperatures. If sufficient pressure is applied to the ice to lower the melting-point to the actual temperature of the ice, it is, of course, capable of melting. An appreciable quantity cannot melt, however, unless heat is supplied from some source at a temperature higher than the pressure melting-point equilibrium. Both the heat capacity of the ice and its thermal conductivity are small and this heat can most readily be supplied from some outside source. If the temperature of the atmosphere is higher than either of the ice surfaces, it could be supplied by conduction from the air. Under these conditions we should expect that the friction of a good thermal conductor would be less than that of a bad one. The friction of a brass ski on cold ice should be *less* than that of an ebonite one.

If, however, the lubricating film is formed by frictional heating, the converse will be true. The frictional heat is liberated at the interface between the sliding surfaces, and if the ski is a good thermal conductor, it will be carried away rapidly and less will be available for surface melting. (See Bowden and Ridler (1936) for an experimental proof of this.) On this view the friction of a brass ski on cold ice should be *greater* than that of an ebonite one. Experiments were made to test these hypotheses, using solid brass and ebonite ski, 1·5 cm. long, 0·5 cm. wide and 0·5 cm. high. The results are shown in fig. 4. The sliding speed was again 400 cm./sec., the load was 130 g., and the temperature of the ice varied from 0 to $-70°$ C. The values again showed some variation and each curve is the average of a number of determinations. The friction of ice on ice is added for comparison.

At temperatures near $0°$ C the frictions of both ski were the same. At lower temperatures, however, the results showed that the friction of the brass was considerably greater than that of the ebonite. The lower the temperature the more pronounced this difference usually became. A similar effect was observed for higher loads (up to 600 g.). At the higher loads μ_k was lower for both substances than at lower loads as in the case of ice surfaces, but the two curves again diverged and at low temperatures the friction of the brass was again greater than that of the ebonite.

Some further experiments were carried out with a hollow ski so constructed that its effective thermal conductivity could be altered while the actual sliding surface was always of the same material. A hollow frame of ebonite was made in the shape of a ski and the bottom was covered with a
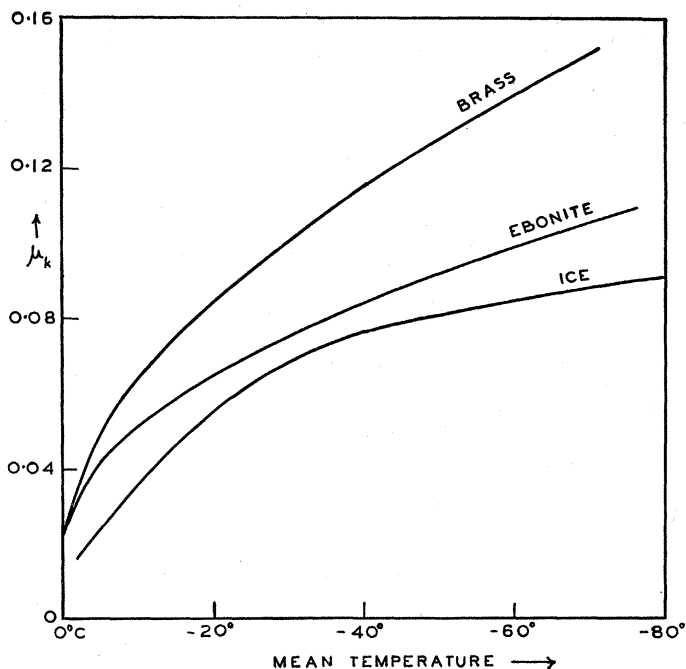

FIG. 4. The influence of temperature on the friction of brass, ebonite and ice sliding on ice.

thin copper sheet. If the air on top of this sheet were displaced by filling the ski with mercury, the thermal conductivity and heat capacity would be considerably increased. The results in Table III show that the coefficient of friction of this ski on cold ice was increased by a factor of 1·3–1·7 when the mercury was added.

TABLE III

Metal	μ_k (no mercury)	μ_k (mercury on ski)	μ_k Hg/μ_k
Copper	0·022	0·031	1·4
	0·027	0·034	1·3
	0·032	0·043	1·3
	0·032	0·054	1·7
Constantan	0·021	0·020	1·0

The thermal conductivity of constantan (0·054) is considerably less than that of copper (0·92), and when the copper was replaced by constantan it will be seen that the addition of mercury had little effect.

These results provide strong evidence for the view that frictional heating plays a large part in the formation of the lubricant water film.

Some measurements were also made with solid brass and ebonite ski heated to 10° C and placed on ice at 0° C. The values of the kinetic friction (sliding speed 400 cm./sec.) are shown in Table IV.

TABLE IV

	Load 200 g.	Load 1000 g.
Brass ski	0·010	0·005
Ebonite ski	0·025	0·010

In this case the heat for surface melting can be supplied by the ski itself since this is above 0° C. It will be seen that the good thermal conductor, brass in this case, now gives the *lower* friction. Under these conditions, when the temperature of the ski is above 0° C, the static friction is also very low, *ca.* $\mu_s = 0·005$.

The friction of hickory ski and the influence of wax.

A small hickory ski, length 4 cm., width 1·3 cm., and thickness 0·2 cm., was made and the lower surface rubbed smooth on fine emery paper. The general behaviour of this ski was similar to that of the other substances, except that μ_k was considerably higher at all temperatures. The ski was then waxed with a thin layer of Sohm's Red Gleitwax, and its friction again measured. The effect of wax on the friction of a small brass ski was also studied, and the results are shown in Table V.

TABLE V

Nature of ski surface	Load 200 g.	
	Temp. −3° C	Temp. −7° C
Unwaxed hickory	0·08	—
Waxed hickory	0·03	0·04
Unwaxed brass	0·030	0·05
Waxed brass	0·025	0·045

The results show that the friction of a hickory surface is reduced to about half its value by a layer of wax. The friction of brass at temperatures near the melting-point is already low so that the addition of wax has comparatively little effect. At these temperatures there is little to choose between brass and waxed hickory as sliding surfaces, but at lower tempera-

tures when the friction of brass becomes high it is probable that the waxed hickory would be considerably better as a low friction ski.

The electrical conductivity of the surfaces during sliding.

If a water film is formed during sliding, we might expect that the electrical conductivity of the surface would change. Two small copper electrodes were set into the bottom of the ebonite ski, and the surface was ground flat and polished. The near edges of the electrodes were separated by a distance of 0·2 cm.

The turntable was cooled down to a low temperature and water containing 0·5 % potassium chloride was poured on to it. This immediately froze to a thin film of ice and the freezing was so rapid that little separation of the salt could occur. The ski was then placed on the stationary surface and a load applied. When the surfaces were cold, the conductivity was less than 5×10^{-7} mho. The surfaces were then set in motion and the surfaces allowed to warm up. The friction gradually fell as the temperature rose, and the value of the friction indicated that surface melting was occurring. The electrical conductivity at first showed little change and the results suggest that the melting was taking place only locally at the isolated points of contact. In the neighbourhood of the melting-point, however, there was a sudden rise in the conductivity to about 5×10^{-5} mho, showing that the water film had become continuous between the two electrodes. This film was still invisible, and under these conditions the friction had the low value of $\mu_k = ca.\ 0\cdot03$.

If we consider the composition of the remelted liquid to be the same as that of the original solution—a drastic assumption—we may make an approximate estimate of the thickness of the film. The specific conductivity of 0·5 % potassium chloride at 0° C is 0·005 mho/cm. The dimensions of the film are $0\cdot2 \times 0\cdot3 \times d$ cm. where d is the thickness. Since the observed conductivity was 5×10^{-5} mho, the thickness d of the liquid film is equal to $ca.\ 7 \times 10^{-3}$ cm. This is a reasonable order of magnitude.

Friction on snow surfaces

Experiments with the model ski.

Experiments were also carried out on snow surfaces. The snow used was finely crystalline snow which had fallen two days previously outside the laboratory. It was placed on the turntable and pressed down with a cold glass plate. It was not possible to run the turntable at high speed under such conditions because the snow was flung off. The experiments on sliding

friction were then made at a sliding speed of about 10 cm./sec. The small ski of brass, ebonite, and hickory were used.

The general behaviour was very similar to that observed on ice, except that the friction was always higher. For example, the friction of waxed hickory on snow was $\mu_k = 0.09$ as compared with $\mu_k = 0.04$ for ice at the same temperature. This higher friction is almost certainly due to the extra mechanical work of displacing and compressing the snow. The friction was again independent of the load, provided this was not too great. At heavy loads the F/W curve became concave to the W axis, as before, showing that the coefficient of friction decreased under these conditions. The effect of velocity was not investigated in detail because of the difficulty of rotating the surfaces at high speeds. There was evidence, however, that the friction decreased considerably at high speeds. The friction was again dependent on the surface temperature of the snow and increased as the temperature fell. Table VI shows some typical results for waxed hickory at different temperatures.

TABLE VI

Temperature, ° C ...	0	− 3	*ca.* − 10	− 40
μ_k	0·04	0·09	0·18	0·4

It will be seen that the friction becomes very high at low temperatures. On the other hand, if the snow was very wet so that the ski was sliding on "slush", then the friction observed, $\mu_k = 0.14$, was higher than that on "dry" snow at 0° C. This is similar to the results obtained on wet ice. It is in agreement with the practical experience that ski run comparatively slowly on very wet snow.

Experiments with real ski.

A pair of hickory ski were waxed with Sohm's Red Gleitwax and lashed together. The load was applied to the centre of the ski by adding rocks of known weight. The experiments were carried out on a level plateau outside the cold laboratory. The snow, which was about two days old, was finely crystalline, and was similar to the snow used in the earlier laboratory experiments. The experimental work was conducted between 10 a.m. and 12 noon, and the sun was obscured by a slight mist. The snow was at 0° C, and towards the end of the experiments some surface melting had occurred.

The force required to pull the ski along was measured with a spring balance. The ski were slid in their own tracks, so that the force of cutting their initial track was not included in the friction. In order to eliminate any errors due to the tracks not being level, the ski were reversed after each measurement, and the values of the friction given are the mean of

these two measurements. Both the static and the kinetic friction were measured, and the results are shown in fig. 5 for a range of loads. The sliding speed for the kinetic friction was about 10 cm./sec.
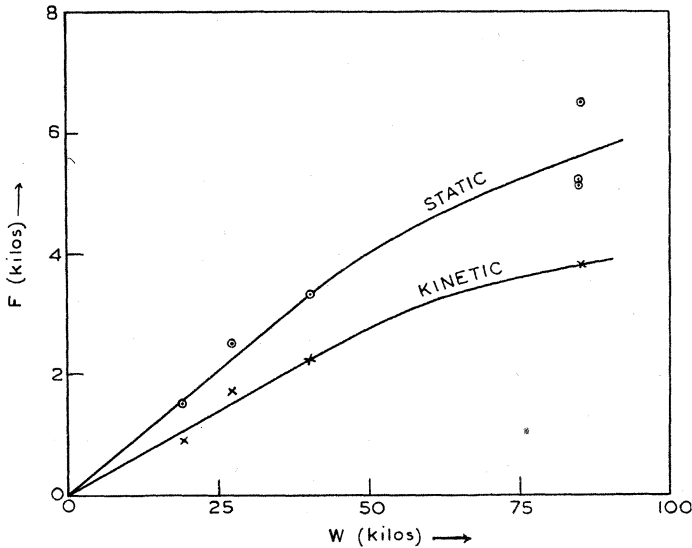

FIG. 5. The F/W relation for real ski on snow.

It will again be seen that Amonton's law is approximately true for both static and kinetic friction. The value of the static friction, $\mu_s = 0.08$, was higher than that of the kinetic friction, $\mu_k = 0.06$. Both the static and kinetic friction were lower than for the model ski on snow in the laboratory. We should anticipate such a result, since the mechanical effect of displacing and packing the snow is relatively greater for the small ski.

After the ski had slid over the track many times and some slight surface melting had occurred, the kinetic friction fell to a lower value, $\mu_k = 0.04$. The static friction under these conditions was also less, and did not differ greatly from the kinetic, $\mu_s = 0.045$.

DISCUSSION

These experiments show that, although the static friction on ice at a few degrees below its melting-point may be of the same order of magnitude as that observed on other solids ($\mu_s = 0.3$ to 0.5), the kinetic friction is very much lower ($\mu_k = 0.02$ to 0.03). This is even lower than the frictions observed between metals lubricated under boundary conditions.

The kinetic friction increases as the ice is cooled down, and on very cold ice at $-140°$ C, it rises to values as high as $\mu_k = 0.12$. At any one temperature the kinetic friction shows approximate agreement with the same general laws observed with other solids. The coefficient of kinetic friction μ_k is nearly independent of the apparent area of contact, it is nearly independent of the load provided that this is not too heavy, and it is not greatly influenced by the speed of sliding provided this speed is not too slow.

The experiments suggest that the low kinetic friction of ice is due to surface melting. Measurements of the electrical conductivity of cold ice surfaces show that the melting occurs only locally at the points of contact, and the water film is not continuous. When the temperature is at, or is very close to $0°$ C, however, the electrical conductivity of the moving surfaces may rise to a high value showing that the water film becomes continuous.

It has often been suggested that the melting is caused by pressure melting. Calculations show however that at low temperatures this is unlikely, even when allowance is made for the difference between the real and apparent areas of contact. On the other hand, some elementary calculations of the amount of frictional heat liberated suggest that this may cause local melting at the points of sliding contact. The experimental results, particularly those showing the influence of thermal conductivity on friction, indicate that this latter view is correct. The results suggest that, although pressure melting may play some part, the low friction observed for rapidly moving solids on ice is due in a large measure to local melting by frictional heating.

The results on snow are very similar to those on ice, except that the friction under any given set of conditions is greater. This is due mainly to the extra work of displacing and compressing the snow.

The observations have an interesting bearing on sledging and ski-ing. No quantitative measurements of the friction of sledges seem to have been published, but there is general agreement that the friction increases at low temperatures. Many arctic explorers, Wright (1924, p. 44), J. M. Scott (1933, p. 273), Cherry-Garrard (1922, pp. 456–7), have recorded that at very low temperatures, -30 to $-40°$ C, the friction between the snow and the runners became so great that the sensation was that of pulling a sledge over sand. Wright, summarizing the conclusions of the Scott Polar Expedition of 1911–13 says:

"Quite apart from any question of the hardness of the snow, however, the surface temperature has an important influence. Our opinion was that the friction decreased steadily as the temperature rose above zero Fahren-

heit ($-18°$ C), the presence of brilliant sunlight having an effect, which was more than a psychological one, on the speed of advance. With air temperatures above freezing point, however, there was some danger that snow would melt in contact with the runners, and subsequently freeze in lumps to increase the friction notably. Below zero Fahrenheit ($-18°$ C) the friction seemed to increase progressively as the temperature fell, as if a greater and greater proportion of the friction were due to relative movement between the snow grains and less to sliding friction between the runner and snow." This steady increase in friction as the temperature of the ice or snow falls is clearly shown in fig. 3. The effect is less marked at very low temperatures, and it is probable that below $-40°$ C very little surface melting occurs under these conditions.

The influence of the thermal conductivity of the sliding body on the friction, shown in fig. 4, is also borne out by practical experience. It will be seen from this figure and from Table III that at low temperatures the friction of a good thermal conductor is considerably greater than that of a poor one. Nansen (1898, pp. 445–6) compared two sledges, one having nickel plated runners and the other maple runners. The temperature was low, the exact value is not given but the mean temperature during that month was $-36·8°$ C ($-34·2°$ F). He found that the friction of the metal was higher, "the difference was so great that it was at least half as hard again to draw a sledge on the nickel runners as on the tarred maple runners".

The thermal conductivity is also important in ski-ing. Nowadays most ski are fitted with brass or steel edges, although sometimes vulcanite or composition edges are used. The friction measurements show that the latter will be faster at low temperatures. If metal must be used, one of low thermal conductivity such as German silver or constantan should be better.

Waxing may reduce the friction of a hickory surface very considerably, and it is probable that the main effect is due to the substitution of a hydrophobic surface for a hydrophilic surface. It is clear that, in addition to this, a very important property of a wax is its ability to adhere to the wood at the local points of rubbing contact. Since the friction of the bare hickory is very high the local removal of wax from the small points of the surface irregularities may cause a comparatively large increase in the friction. The ability of the wax to flow over the surface and to repair the local damage may also be an important factor. In practice waxes differing very widely in physical and chemical properties are used under different snow conditions. No systematic investigation of the friction and behaviour of different waxes has so far been made.

SUMMARY

Experimental studies of the friction on ice surfaces have shown that the low frictions observed at temperatures near the melting-point were due to lubrication by a thin water film at the points of contact between the sliding surfaces. The coefficient of kinetic friction was found to be independent of the load, apparent area of contact, and speed of sliding over a certain range. When the temperature of the ice was decreased, the friction rose markedly as the water film became more difficult to form. Using ski of various materials, it was observed that the friction depended very largely on the thermal conductivity of the ski. This result suggested that frictional heating played a large part in melting a water film during sliding; it had been previously considered that pressure melting was alone responsible for the formation of this water film.

Experiments with miniature and real ski on snow surfaces showed that the same general laws were obeyed as on ice surfaces. The higher frictions obtained on snow were attributed to the extra work done in displacing and compressing the snow crystals.

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