DYNAMICAL ISSUES: SEMIMAJOR AXES

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Outline

• Long term stability of planetary systems
• A few points on the stability of the solar system
• Known exo-planets: semimajor axis distribution
• Multiple exo-planet systems — orbital diversity
• Final stages of planetary orbital evolution — origin of orbital diversity
• Conclusions/Speculations

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“Planetary architecture is determined by long term stability”
\[ \implies \) (roughly) well separated orbits

- How to quantify “well separated”
  - depends upon \( m, a, e \)
  - stability criteria for 1 planet system
    - Hill stability: \( |\Delta a| / a_p > 2.4 (m_p / m_\star)^{\frac{1}{3}} \approx 3R_H \)
    - Resonance overlap: \( |\Delta a| / a_p > 1.5 (m_p / m_\star)^{\frac{2}{7}} \)
    - Eccentric planet: \( r \ni (q - 3R_H, Q + 3R_H) \)
  - secular stability in 2 planet systems
    (see poster)

- No global stability criterion for few-body systems
  - few-body phase space is an intricate mix of chaotic orbits and stable orbits
  - Stability is not monotonic function of separation

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Resonance overlap in the 3 body problem
planar, circular, restricted

- KAM curves — global stability
- resonance: islands of stability in chaotic sea

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Chaos in Solar system outer planets

from Murray & Holman 1999

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A solar system in ‘nearby’ phase space – long term stable (i.e. no orbit-crossings) but not as ‘quiescent’ as the actual solar system
Two classes:

- hot jupiters – *in situ formation or migration?*
- ‘normal’ giant planets – *late(r) formation, therefore little or no migration?*  
  – Implicates diversity of protoplanetary disk (gas) masses.
Multiple planet systems

- diverse architectures!
- resonant planets
  orbital migration in gas disk?
- regular systems
  low eccentricity orbits
- irregular systems
  high eccentricity orbits

Observed diversity suggests that giant planets do not form in well-separated orbits. Orbital evolution owing to planet-planet/debris (chaotic) interactions leads naturally to diverse outcomes.
Final stages of planetary system formation

gas-free environment
not to forget the small bodies!

Two extrema of evolutionary paths can be identified:

- **planet-planetesimal disk interactions**: migration as in our solar system — planet eccentricities remain damped while semimajor axes spread out
  figure: Hahn & Malhotra 1999 ⇒ p. 9

  Evidence in the remains of solar system debris: Kuiper Belt, Oort Cloud

- **planet-planet interactions**: if the debris disk mass is insufficient, then planet eccentricities will not remain damped, crossing orbits will develop and lead to mergers, scattering, ejection
  figure: Ford et al 2001 ⇒ p. 10
Planet migration via planetesimal debris disk

from Hahn & Malhotra 1999
Planet-planet scattering

from Ford et al 2001
Summary: A flowchart of planetary system formation

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