*The*

# *KILOGRAM*

*Definition, Realisation,*
*and its Replacement*

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*Abstract*

The unit of mass stands alone among the fundamental units of the Système International (S.I.) in that its definition still rests on an artefact standard. Within a historical context, the methods used in the construction and comparison of such standards are discussed, the shortfalls of such definition are indicated, and the progress of two techniques which may in time replace the artefact with the fixing of fundamental constants is reviewed.

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# <u>The kilogram - Definition, Realisation and its Replacement</u>

*Three Forms of Measurement Standard*

Among disciplines of study, physical science is marked out as one in which the order and beauty observed in the universe is attempted to be appreciated in a quantitative form. In order for this to be done it is necessary to measure the quantities involved: and this requires a system, and hence standards, of measurement. Of course, much physics can be done with scaling laws and relative measurements - and this is well attested by the large number of graphs appearing in published journals with "Arbitrary Units" or "Relative Units" labelling at least one of the axes - and for these the Système International is irrelevant. However the crippling effect of having no internationally agreed standards for reference on the reproduction and validation of experiments, as well as on the dissemination of the information discovered by them, is hard to overestimate. Given that a system of measurement is needed, it is necessary to consider the various methods by which a given unit may be "standardised".

The preferred method is to define the unit in terms of other units already in existence. Thus it is natural to measure area in terms of "metres squared" and velocity in "metres per second". Indeed, it would be highly inconvenient to do anything else. Nevertheless, great care must be taken to ensure that definitions do not become cyclic. A cautionary tale along those lines has been related by Cohen[1] in which a retired sea captain who lived in a remote part of the isle of Zanzibar had a custom to fire a cannon at noon. When asked by a visiting friend how he set his chronometer, he replied that there was a horologist's shop in the town, and that he set his clock by the timepieces on display in the window. The friend subsequently asked the same question of the horologist, and received the reply "There is a retired sea captain on the cliff who fires a cannon at noon..." Thus although there was no doubt as to the consistency of the measure of time on Zanzibar, the positions of the hands on every single clock had little to do with the time of day. Petley puts this situation more concisely as "equivalent to attempting to lift a bucket while standing inside it."[2]

A second method is to define the unit in terms of a phenomenon common to all observers. Thus the original definition of the second in terms of the length of a day, and the mediaeval definition of the inch "three barleycorns dry and round", fall into this

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[1] E.R. Cohen, K.M. Crowe and J.W.M. DuMond, 1957, *Fundamental Constants of Physics* (Wiley)
[2] B.W. Petley, 1985, *The Fundamental Physical Constants and the Frontier of Measurement* (Hilger)

category. Such definitions usually require two parts - the fixing of a fundamental constant in the system of measurement, and statements concerning the way in which the measurement is to be made to reduce the error in reproducing it. In the case of the second we have the fixing of the rotation rate of the earth as 1/(24*60*60) revolutions per second, which determines the magnitude of the unit of time. In the case of the inch, the length of the barleycorn is fixed, but to lower the error of measurement we are required to avoid both excessively long, thin corns, and also those swollen by exposure to water. The barleycorn is a good choice, since they were plentiful in every village of the nation involved, and thus "conveniently realisable to all concerned" to use modern metrological terminology.

A shortcoming of this type of standard (especially during the middle ages) was that the physical "constants" chosen were either not particularly constant, or were difficult to measure at the time. The standard deviation of the length of barleycorns is a good fraction of the length itself. The most precise of ancient definitions, therefore were of the third type - they were artefact standards. In other words, some object was constructed to define the unit, and from then on was used as sole and absolute authority on matters of calibration. This method was used to great effect by the Egyptians in the construction of the pyramids, the side lengths of which are equal to 0.05%. This phenomenal accuracy was obtained by defining the Royal Cubit (approximately 524mm) as the length of a black granite rod, which was formed the apex of a hierarchy of standards, at the base of which resided the myriad wooden cubit sticks used daily in construction. The principal standard was designed purely for stability, and its complete unsuitability as a construction tool was no inconvenience. Indeed, once an artefact standard has been constructed, it is imperative that it is treated kindly. If a measuring rod gets broken on the building site it is no great loss. If the principal standard of measurement is similarly destroyed, then continuity of the unit is uncertain.

For the reasons given above, it is no surprise that by the 18th century, the only unit of any importance not defined by an artefact was the second - for which no artefact can be constructed anyway. In the time since then, and especially in our own century, there has been a trend away from this back to standards based on fundamental properties of the universe. As examples from 1900-1940, the metre was measured in terms of the wavelength of a spectral line as early as 1902 by Michelson, and the unit of current was defined both in terms of electrochemical deposition of silver and the electromagnetic force between two parallel wires carrying current in the same direction - the latter definition holding to this day. This definition of the ampere is also of interest because it includes the caveats that the wires be of "negligible circular cross-section", be "placed

1 metre apart in vacuum" and be "of infinite length".[3] These particulars are, I suppose, simply the modern equivalent of the "dry and round" specification of the barleycorns in the mediaeval inch. This trend is all to be encouraged as scientific progress in the last 120 years or so has shown an abundance of numbers which have particular significance for the world in which we live. A system of measurement in which the speed of light, the magnetic permeability of free space, the Planck constant, the electronic charge and the electronic mass were fixed would have much to commend it - its basis being the most universal and ubiquitous quantities that we know of. Also this avoids the problems associated with the limited accessibility and localisation of an artefact standard, and the dire consequences of it being involved in any accident.

There is, however, a fly in the ointment. Of the fundamental units in the Système International, only one has always been, and still is, defined in terms of an artefact. It is the kilogram - the standard of mass, and as such, the basis of all mechanical measurements undertaken in science, engineering and commerce.

*The Definition of the kilogram*

The kilogram, along with the whole metric system, was born in the overhaul of the French outlook that accompanied the Revolution. In measurement, as in other walks of life, old standards were abrogated and completely independent replacements were set up. In contrast to many other changes, such as dating events from the Revolution instead of the birth of Christ, which were subsequently repealed; the metric system has remained. No doubt this is at least partly due to its superiority over its predecessor in being entirely decimal, and in having units of different quantities that were clearly related. The new units, as approved by the National Assembly of France in 1795, used for a foundation a standard of length, which also gave its name to the system as a whole. Indeed to this day, when nations agree to take part in the S.I. and are presented with copies of the various standards, they are still said to become "signatories of the Metre Convention".

The metre was defined as 1/ 10 000 000 part of the distance from the North Pole to the Equator on the Parisian meridian, and a geodesic survey team was dispatched to measure this length as accurately as then possible. On their return, a standard rod with the cross section of an "X" was cast, based on their observations, to be the first metre rule. It was presented to the National Assembly in 1799, and was approved as the

[3] *S.I. The International System of Units*, 1986 (HMSO)

artefact standard of the new unit of length. At the same time the unit of mass, the kilogram, was intended to be that of $1dm^3$ of water at its highest density.[4] As with the metre, an artefact standard was required, and a platinum cylinder was cast for this purpose. It became known as the “Kilogram of the Archives”.

By the time a century had passed, many other countries had adopted the Metric System, and it became clear that the administration of the units should be passed to an international committee. The Conférence Générale des Poids et Mesures (or CGPM) was set up with the duty to oversee the metric system, and at its first meeting it ratified the standards of measurement of all significant units. As far as mass was concerned, it had been decided that improved stability would result from the casting of a new standard from a platinum alloy containing 10% iridium. Three cylinders of this material were cast to be as close in mass to the Kilogram of the Archives as possible.[5] The third mass was measured to be the nearest, and as such it was sanctioned by the first CGPM with the words “This prototype shall henceforth be considered as the unit of mass”.[6] Of the definitions ratified at the time, this is the only one that is still current.

The international prototype kilogram is shown in Figure 1a, and is kept at the Bureau International des Poids et Mesures under three bell-jars, the outer of which is a vacuum vessel. The standard itself is a cylinder with both diameter and height nominally equal to 39mm. The choice of a cylindrical shape is significant. It is desirable to minimise the surface area of a mass standard, since it is on the surface that many of the potential sources of inaccuracy arise. Hollows provide refuges for dust, while protrusions cause enhancement of electrostatic field, and hence of corrosion through electrochemical reaction. The ideal shape, a sphere, was rejected because of difficulty of machining the object at the time. Also spheres are difficult to support uniformly, and they have the very unwelcome property of being able to roll. Thus a cylinder was chosen as the best alternative. The two edges are rounded to reduce the effects of increased corrosion rate in regions of small radius of curvature.

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[4] Encyclopaedia Britannica, Macropaedia, *Measurement Systems*

[5] C.H. Page, P. Vigoureux, 1975, *The International Bureau of Weights and Measures 1875-1975* (US Department of Commerce)

[6] *Comptes rendus des séances de la CGPM*, as quoted in *S.I. The International System of Units*, 1986 (HMSO) which is sanctioned as an accurate translation of the French original by the director of the National Physical Laboratory.


(a)

(b)

**Figure 1: Prototypes of the standard of mass. (a) The International Prototype kilogram, kept at the Bureau International des Poids et Mesures at Sèvres near Paris. (b) Kilogram 18, which serves as the United Kingdom standard, and is kept at the National Physical Laboratory in Teddington, Middlesex.**

As the kilogram is defined in terms of an artefact, it is necessary for a chain of propagation of the unit to exist to ensure that daily measurements undertaken in business or research can be traced back to the International Prototype, even though it is only weighed a few times in one occasion every 20 years or so. To do this a series of copies of the prototype were made shortly after its own construction. A few of the copies are maintained at the Bureau International for comparison with the International Prototype and for the more frequent weighings which are necessary if the laboratory is to provide practical calibration services to science and industry. The other copies are given to each of the member states of the Metre Convention for use as national standards of mass. The United Kingdom standard, shown in figure 1b, is kilogram number 18, and is kept at the National Physical Laboratory in Teddington, Middlesex. The national standards laboratories possess further sub-standards for daily calibrations, and the national standards themselves are sent to Paris periodically for comparison with each other and with the International Prototype. While efficient, this chain does lead to uncertainty in the relative magnitudes of the different nations' mass standards. Thus although with current weighing technology, it is reckoned that balances can perform measurements accurate to 1 part in $10^{11}$, and that the International Prototype is maintained to an accuracy of order micrograms; the National Physical Laboratory only certifies weights in their calibration procedure to a milligram tolerance.

*Kilogram Balances*[7]

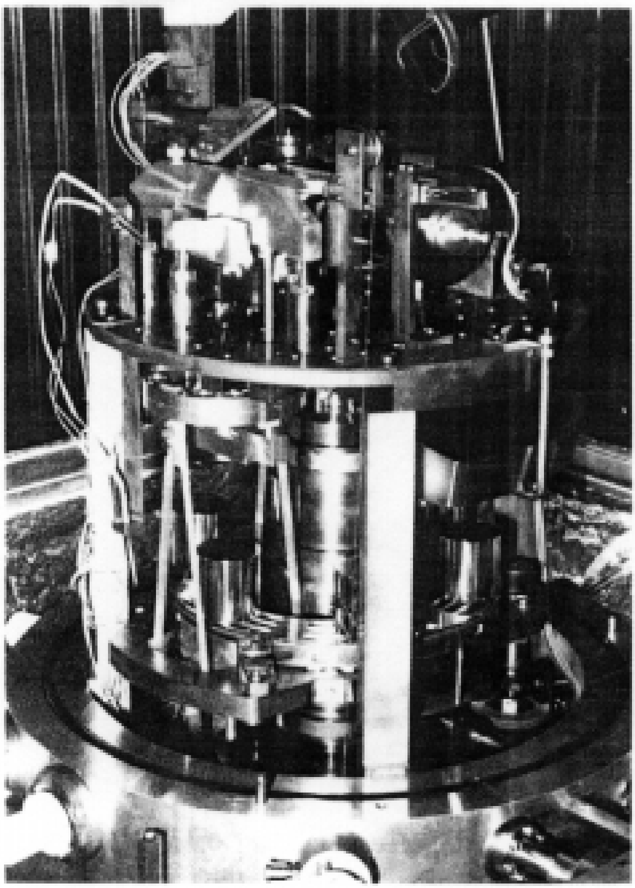

**Figure 2: The single arm knife edge balance developed at NBS now used at BIPM. The masses for comparison are clearly visible on the turntable in the lower stage.**

The principal method of comparing two masses has always been by means of a balance, and historically this measurement has always been conducted in air. Thus to ensure continuity it is necessary to continue to operate these balances at atmospheric pressure. This presents difficulty in comparing weights made from different materials, and hence having different density, since the effects of both buoyancy in air, and the adsorption of air molecules onto the surface of the weight need to be taken into account. For this reason all of the major kilogram masses are constructed from the same alloy as the prototypes. Even then, the task of constructing a sufficiently accurate balance is extremely difficult given that the arm lengths must vary by no more than an atomic radius. It is thus necessary to make the measurements in temperature controlled enclosure. One of the principal remaining causes of arm length change is the sudden

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[7] For history and technology of balances, refer to C.H. Page, P. Vigoureux, 1975, *The International Bureau of Weights and Measures 1875-1975* (US Department of Commerce) for history up to 1975, and

relaxation of strains brought about by differential thermal expansion at points where different materials are joined together. This is reduced by forming the components out of a single material, but even then care must be taken to ensure that unstable points of high stress are avoided.

Until the 1960s the most accurate method of comparing standard masses used an equal arm balance suspended on a knife edge, with the two pans suspended by knife edges from the balance arm. A more precise technique developed in the 1960s at the United States' National Bureau of Standards, involved a "single arm balance" in which the masses were compared one at a time with a platinum iridium counterweight. Such a device is shown in figure 2, with a schematic sketch in figure 3. In this type of balance it is imperative that the atomic environment of the pivot remains unchanged between the weighings.

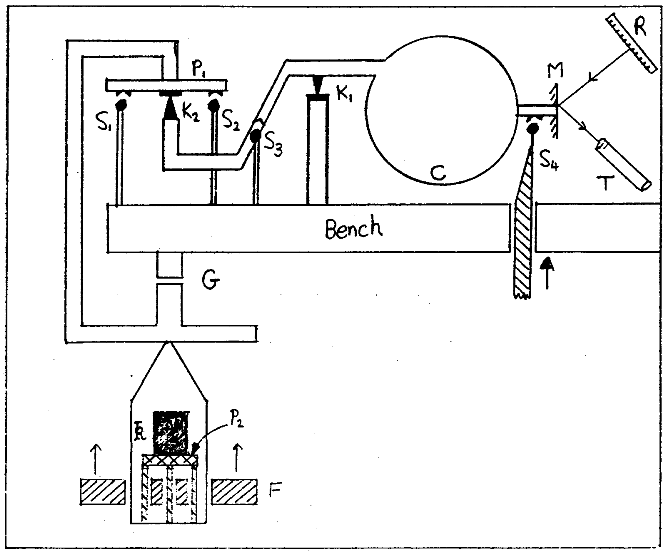


**Figure 3: Schematic diagram of Single Arm Balance. See text for explanation of operation.**

When the balance is not needed, it is placed in the rest position by raising support S4 to its upper position. The counterweight C, and arm are held on S4 and S3, and the weighing platform P1 is held on supports S1 and S2. The precision knife edges and flats K1 and K2 are thus not worn down when the balance is not in use. Before weighings are performed, the balance is made ready by lowering support S4, allowing the balance arm to rock freely. The platform now rests on knife edge K2, and the

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to T.J. Quinn, C.C. Speake, R.S. Davis, 1986, *A 1kg Mass Comparator Using Flexure-Strip Suspensions: Preliminary Results* Metrologia **23** p87 for more recent developments.

balance-point is at knife edge K1. The deflection of the beam is read from the calibrated scale R which is viewed, via mirror M, through telescope T.

This enables kilogram **k** to be weighed as it stands on its plinth P2. Once the scale has been read, the kilogram is lifted on its plinth by raising frame F. This can then be slid out of the cradle, and a new mass inserted, and lowered. During this whole process the knife edges are kept in virtually the same position because when the kilogram is lifted the gap G, which is of the order of microns, closes and this prevents further movement of the balance arm. The different kilograms are weighed, and also weighed with each others' plinths so that the masses of the plinths can be taken into account. In addition, small weights of known mass (of order milligrams) are placed on the plinths in addition to the kilogram so that the scale which measures beam deflection can be calibrated in terms of mass. The mass of these small weights need be known to a much smaller relative accuracy, and thus the determination of their masses does not pose a significant problem. Using this type of balance accuracies of 1.4μg are achievable, although care must be taken to avoid excess systematic error in the motion of the rest point which can be as high as 7μg. This type of systematic error is virtually unavoidable in knife edge balances in which the arm is free to oscillate - although in 1984 a balance at the Japanese National Research Laboratory of Metrology was reported to be able to compare standard kilograms to an uncertainty of 0.3μg. This balance uses knife edges of sapphire and flats of ruby, and is fully automated. The automation of balances, whether by electronic or remote mechanical means greatly reduces the errors in measurement, removing the effects of the motion of the operator and the air currents so generated.

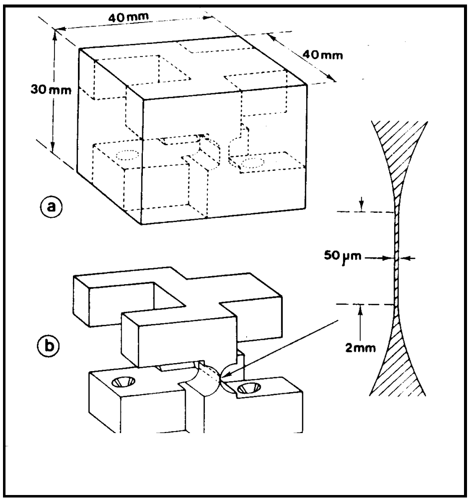


**Figure 4: Cu-Be Flexure joint for use in mass comparator balance**

An improvement in recent years has come with the development of balances which replace knife edges with flexure joints, usually made of a copper-beryllium alloy. The pivot point is thus now fixed, and this source of error obviated. Such a joint is shown in figure 4, and a balance design incorporating these in figure 5. By means of these the BIPM now is able to state that weight comparisons are accurate to 1 part in $10^{11}$. Despite these phenomenal accuracies for a weight measurement, for the reasons given above there is much active research being undertaken to replace the artefact standard with the fixing of a fundamental constant. The need is all the more urgent now that the ampere is no longer realised directly, but rather by the equivalence of the mechanical and electrical watts. Thus any uncertainty in the definition of the kilogram is fed into all the electrical units, and hence into all electrical measurements.

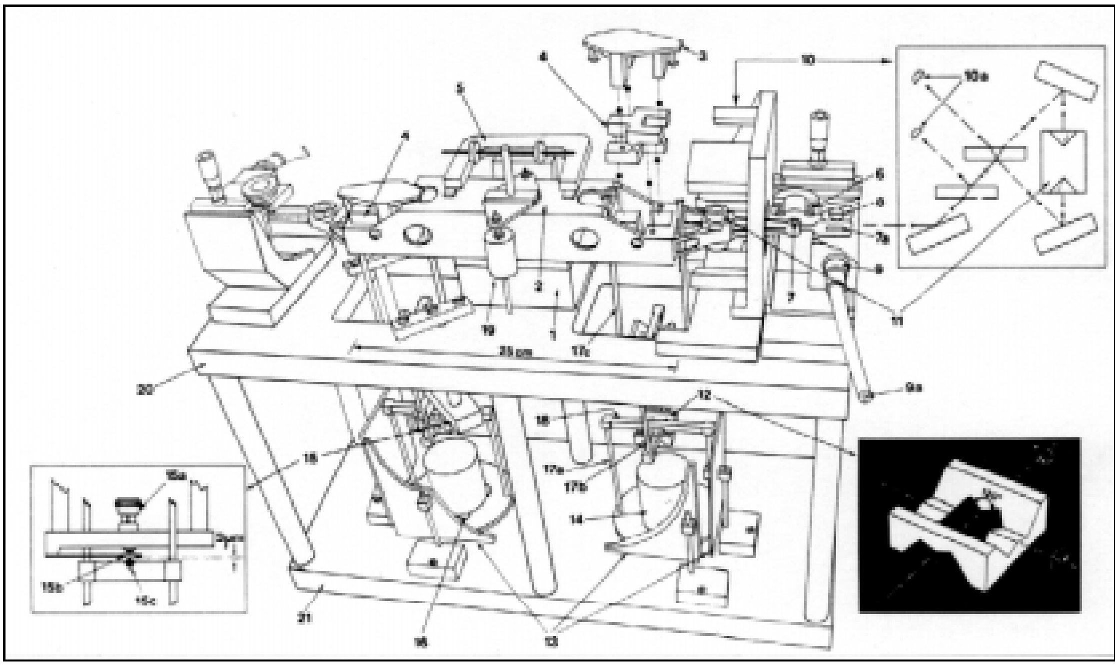

**Figure 5: Balance using flexure joints as in figure 4.**

1 Central pivot support block, 2 aluminium alloy balance beam, 3 pan-suspension mounting bracket, 4 end flexure strip, 5 locking mechanism, 6 servo-control coils, 7 a slotted screen, 8 split photodiode, 9 and 9a collimator and optical-fibre cable, 10 and 10a interferometer and photodiodes, 11 double cube-corner reflector, 12 central component of knife edge assembly, 13 parallel plate air dampers, 14 1kg load, 15 adjustable pan stop mechanism, 15a micrometer for adjustment of stop, 15b hardened steel disc, 15c Cu-Be ball, 16 one of the three holes in pan allowing passage of pins attached to lifting device, 17a,b,c three sections of pan suspension, 18 position where a mechanism (not shown) can place a sensitivity calibration mass, 19 one of the two movable masses used to adjust the centre of mass of the balance beam, 20 and 21 upper and lower base plates.

*An Electrical Redefinition for the kilogram*[8]

In the past twenty years, two effects have become well understood that have radically improved the accuracy of realisation of electrical units. The first of these is the

[8] B.P. Kibble, 1991, *Present State of the Electrical Units*, IEE Proceedings A **138** (3) p187

Josephson Effect, in which a Josephson junction essentially forms a frequency to voltage converter (see figure 6). When a thin insulating layer joining two superconductors is subjected to microwave radiation of frequency $\nu$, a d.c. voltage of magnitude $V = h\nu / 2e$ is established across the junction. Given the high accuracy of the realisation of the second, this has the potential to give very precise voltage determinations.[9] Indeed, it was precisely this method that initially indicated the presence of systematic errors of 8 parts per million in the realisation of the ampere with a current balance.

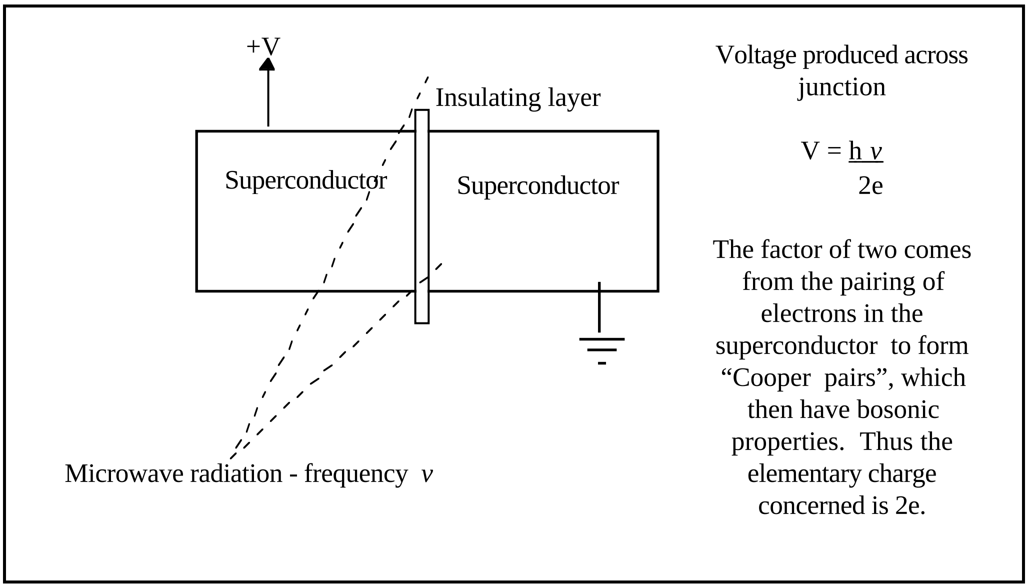


**Figure 6: The Josephson junction and its use as a voltage standard.**

The second phenomenon is that of the Integer Quantum Hall Effect. The situation is shown schematically in figure 7a: the Hall voltage is measured across a very thin piece of semiconductor which carries an electric current while present in a strong magnetic field at low temperatures (typically a few tesla at 0.3K). Both the voltage drop along the semiconductor and the "Hall resistance", that is the ratio between the Hall voltage and the current passing through the device, have a complex dependence on the magnetic field, as shown in figure 7b. For certain values of the magnetic field, the voltage drop falls almost to zero, and at these points, the Hall resistance is given by $R_H = h / p(B)\ e^2$, where p(B) is an integer.

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[9] NPL, 1987, *Measurement Services: Direct Current and Low Frequency Electrical Measurements* (HMSO)

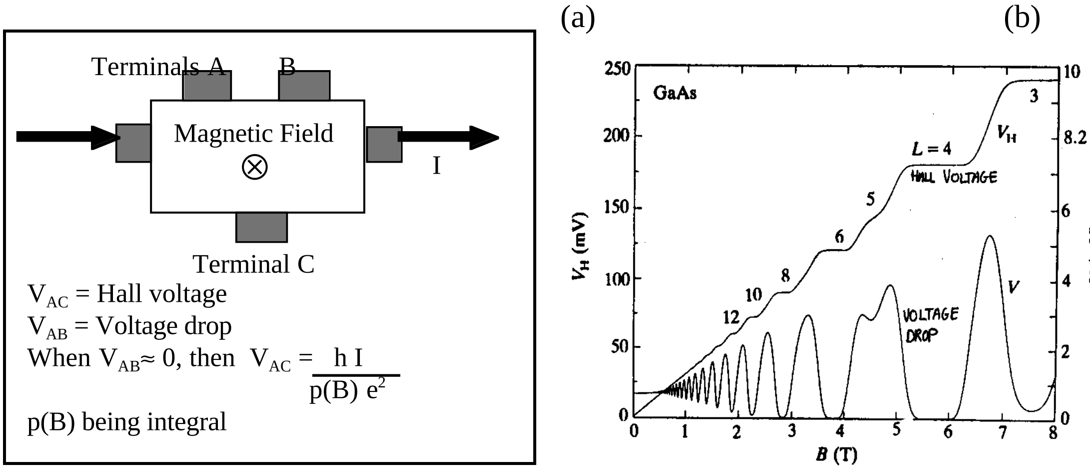


**Figure 7: The Integer Quantum Hall effect. (a) Geometry of apparatus, (b) dependence of Hall voltage and voltage drop on magnetic field for a fixed current.**

Currently the Josephson effect is used in nearly all national metrology laboratories for the day to day maintenance of their voltage standards, and it and the Hall effect have given us the most accurate values to date for the Planck constant (h) and the electronic charge (e). One possible method for re-defining the kilogram involves fixing these two fundamental constants. The Volt and Ohm would then be independently defined, and hence so would the electrical watt. Given that the mechanical and electrical watts must be equivalent, and that the metre and second are well defined, this gives us a procedure for measuring mass without recourse to an artefact.

The device used for equating the measurements of electrical and mechanical power is known as a "Watt balance", and its principles of operation are shown in figure 8. The apparatus consists of a precision balance which is horizontal when the mass-holder H is empty and there is no current in the coil C. In the first part of the experiment, a standard kilogram is placed in the holder, and the balance brought back to the horizontal by passing an appropriate current through the coil and standard resistor R in series. A voltmeter V measures the potential difference across the resistor. The equivalence of electrical and mechanical virtual work is then used to derive the equation of balance. Given the equation for gravitational potential energy $E_{mech}=mgx$, we have that the force (the weight) of the mass is of course $-dE_{mech}/dx = -mg$, where x gives the vertical position of the coil and is taken as increasing upwards. The electromagnetic force exerted by the coil, which must be equal and opposite, and considering that the current is constant, is given by $-dE_{elec}/dx = d(V\ I\ dt)/dx = -d(\Phi\ I)/dx = -I\ d\Phi/dx = -I\ \partial\Phi/\partial x$, where Φ represents the flux linking the coil. Thus the equation for balance is $mg = -I\,\partial\Phi/\partial x$, where $I = V/R$.

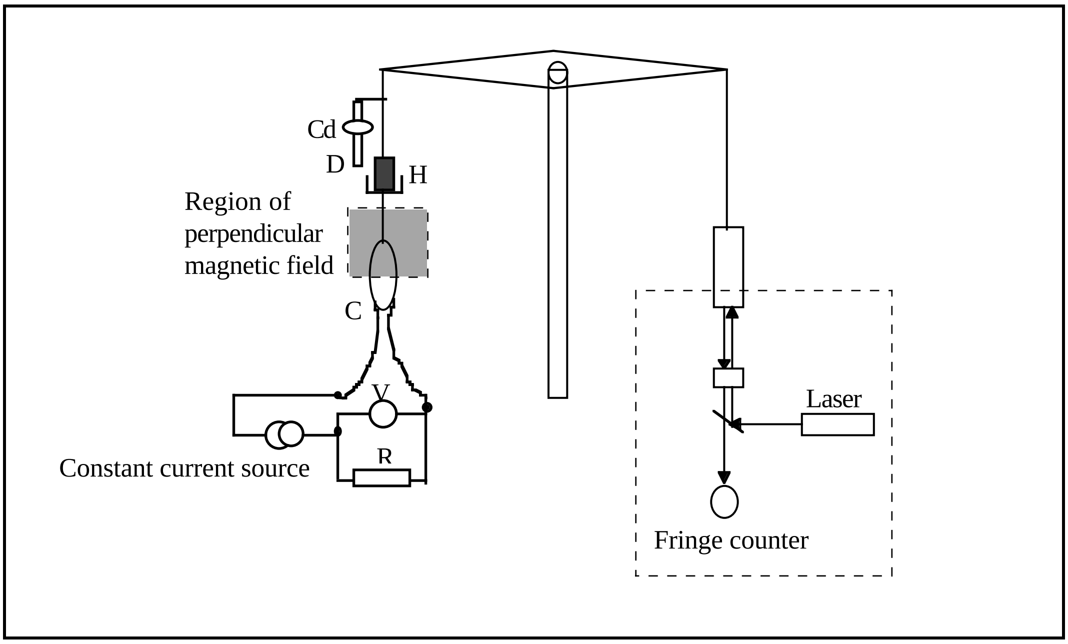


**Figure 8: The Watt balance for determining the kilogram in terms of the volt, ohm, metre and second.**

In the second part of the experiment, the kilogram is taken off the mass holder, and the constant current source is removed from the circuit, the coil now being connected directly across the resistor. The balance is now prone to swing, and as it swings its speed is monitored by means of the optical interferometer. This speed is kept constant by means of electromagnetic damping (via the induction of eddy currents in the non-magnetic piece D by coil Cd). As the balance passes the horizontal position, the voltage V' is measured. Hence $V' = I'R = -d\Phi/dt = -u\,\partial\Phi/\partial x$, where $u = dx/dt$ is the speed of the coil. Combining the two equations, we find that $mguR = V\,V'$. Thus if the voltages can be measured using the Josephson volt and the resistance via the Hall ohm, then given that the metre and second (and hence u and g) are well defined, then the mass can be obtained from the formula. Using this technique, the electrical and mechanical watts are linked with less than 1 in $10^8$ uncertainty.[10,11]

[10] B.P. Kibble, I.A. Robinson, J.H. Belliss, *The New NPL Moving-Coil Watt Balance - A Progress Report*, Private Communication

[11] B.P. Kibble, I.A. Robinson, J.H. Belliss, 1990, *A Realisation of the SI Watt by the NPL moving-coil balance*, Metrologia **27** p173

*Atomic Redefinition of the kilogram*

A more intuitive redefinition for the kilogram would be based on the mass of an atom or a more elementary particle. This is requires the precise determination of the Avogadro number, and also the mass ratio of the carbon 12 atom to the elementary particle chosen as standard. For this reason, this research work is often referred to as the "Avogadro Project".[12] The task is aided considerably by the wealth of experience gained in the last 40 years in growing silicon crystals for the semiconductor industry, and in studying the properties of these crystals. It is now possible to grow a perfect silicon crystal large enough that a kilogram sphere may be fabricated from it, with the departure from sphericity of the order of tens of nanometres.[13]

In order to redefine the kilogram, four precise measurements need to be made: the molar mass of silicon in terms of the mass of the elementary particle chosen, the lattice spacing in silicon, the diameter of the sphere, and the weight of the silicon sphere in comparison with the platinum iridium kilogram standard.

The molar mass determination is made by high precision mass spectroscopy.[14] As such this involves no new technique to be developed, but in order to achieve the necessary accuracy, effects such as the time dependent adsorption of the different isotopes on the wall of the region of the spectrometer connecting the molecular leak to the ion source need to be considered.[15] It has also been proposed to use prompt $(n,\gamma)$ spectroscopy to determine the isotopic abundances.[16]

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[12] P. Seyfried, P. Becker, 1994, *The Role of $N_A$ in the SI: An atomic path to the kilogram,* Metrologia **31** p167

[13] A. Leistner, W. Giardini, 1992, *Fabrication and Testing of Precision Spheres*, Metrologia **28** p503

[14] For an overview see "Mass Spectrometry" on p1087 of G.W.F. Drake (Ed.), 1996, *Atomic, Molecular, and Optical Physics Handbook* (AIP)

[15] R. Gonfiantini, P. De Bièvre, S. Valkiers, P.D.P. Taylor, 1997, *Measuring the Molar Mass of Silicon for a Better Avogadro Constant: Reduced Uncertainty*, IEEE Trans. Instrum. Meas. **46** (2) p566

[16] S. Röttger, A. Paul, U. Keyser, 1997, *Prompt (n,γ)-Spectroscopy for the Isotopic Analysis of Silicon Crystals for the Avogadro Project*, IEEE Trans. Instrum. Meas. **46** (2) p560

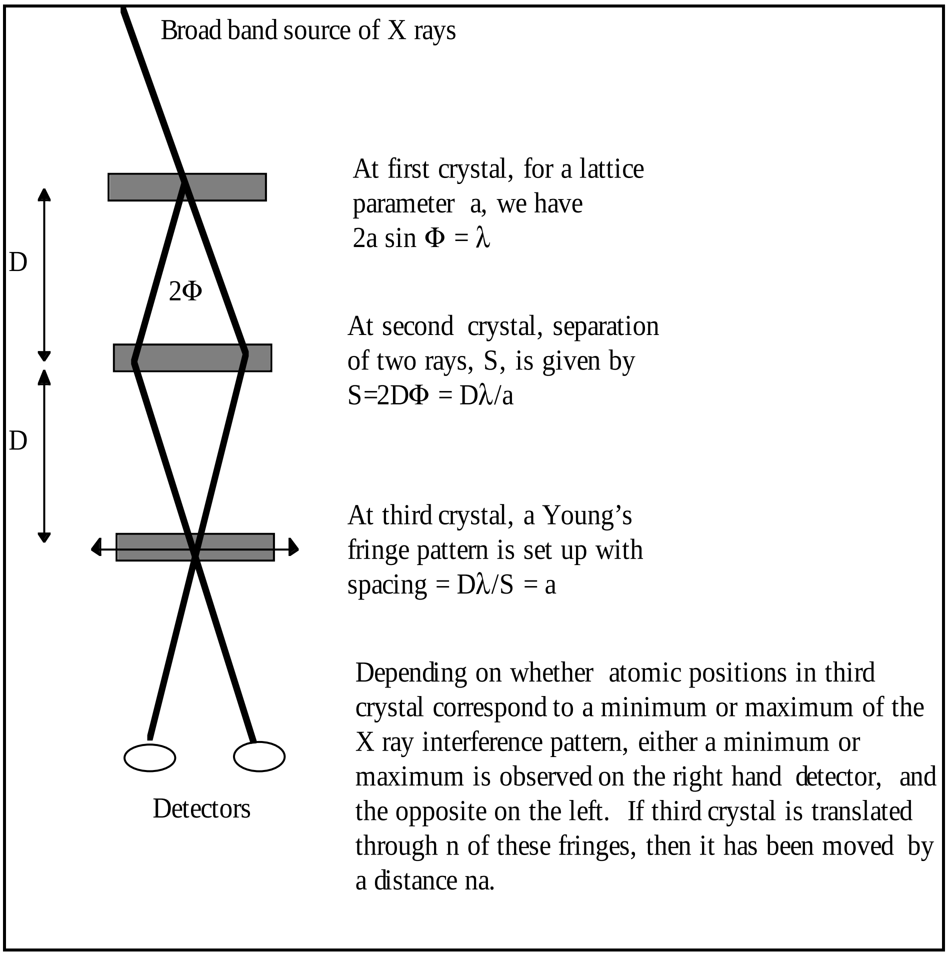


**Figure 9: The principle of the Bonse-Hart Interferometer.**

At first sight, the determination of the lattice parameter of silicon would require high precision X-ray spectroscopy, and as such would require a highly monochromatic beam of well known frequency. In practice X-ray wavelengths are not known to the accuracy required, and so such a method is not suitable. A solution is found, however, in the interferometer of Bonse and Hart[17], in which lattice spacing is determined using a broad band source, and no knowledge of the X-ray wavelength is required.

In this spectrometer, shown diagramatically in figure 9, three identical silicon wafers (cut from the same crystal) are held parallel[18] to each other, and with equal spacing. The third crystal is translated with respect to the others, and cosine-squared fringes are observed on both detectors. The number of fringes passed gives the distance moved in

[17] U. Bonse, M. Hart, 1965, *An X-Ray Interferometer,* Applied Physics Letters **6** (8) p155

[18] In practice, the parallelism is achieved by cutting away a thickness D from a single crystal, leaving two parallel plates (distance D apart) still held parallel by a backbone of the original boule which is permitted to remain. The moving crystal is made parallel to the other two by placing it on a Bragg goniometer and rotating it until the required orientation is obtained.

lattice parameters. The magnitude of the translation is measured using an optical interferometer, so that one optical fringe is observed for every $\lambda/2$ translation of the crystal, where $\lambda$ is the wavelength of the optical radiation. Thus the crystal is translated, and if $n$ X-ray fringes, and $m$ optical fringes are observed, then the lattice parameter is given by $a = \lambda m / 2n$. A more detailed diagram of the experimental apparatus is given in figure 10, and using this the lattice spacing of silicon was measured[19] as $(1.920154 \pm 0.000002) \times 10^{-10}$m.

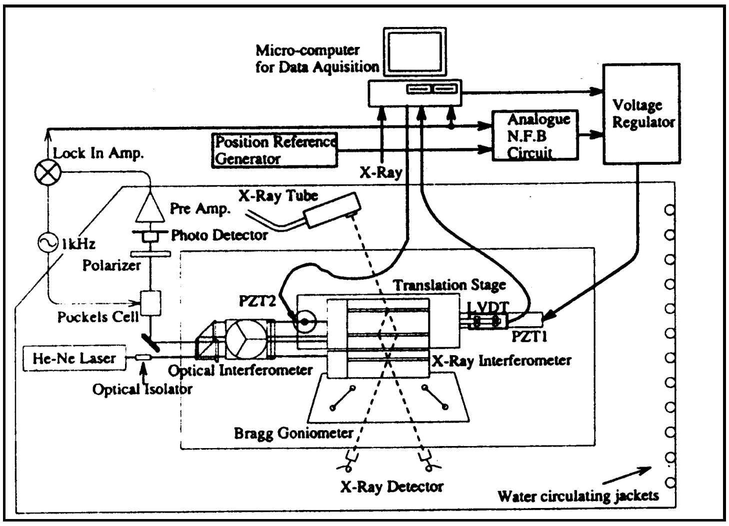


**Figure 10: Apparatus for determining the lattice parameter of silicon using a Bonse-Hart interferometer.**

The third challenge is to measure the diameter of the silicon sphere. This is done using the techniques of optical interferometry as shown in figure 11. The sphere is placed inside a Fabry Perot etalon, and the etalon scanned so that fringes are produced at the detectors. The fractional order of interference is determined from the intensity detected. In practice, the laser beams are chopped by an acousto-optic modulator, at a frequency of 175 Hz. This same reference is fed to the lock-in amplifiers which process the signal from the detectors. The difference in the phases between the two detectors can be used to infer the fractional order of interference. The integer part of the order of interference is calculated by the method of exact (or excess) fractions. In other words, two lasers of different frequencies are used, and given that the fractional order of interference is known for both frequencies for an approximately known range of positions of the etalon, simultaneous algebra can be used to infer the integral number of fringes.[20] This gives the distances between the etalon flates and the sphere, and hence from a

[19] K. Nakayama, H. Fujimoto, M. Tanaka, K. Kuroda, 1993, *Silicon Lattice Measurement with an Improved X-Ray/Optical Interferometer*, IEEE Trans. Instrum. Meas. **42** (2) p401

[20] For fuller explanation, see §15.3 of M. Françon, 1966, *Optical Interferometry* (Academic)

knowledge of the etalon spacing, the sphere diameter is obtained by subtraction. A series of measurements give an indication of the sphericity the sample.

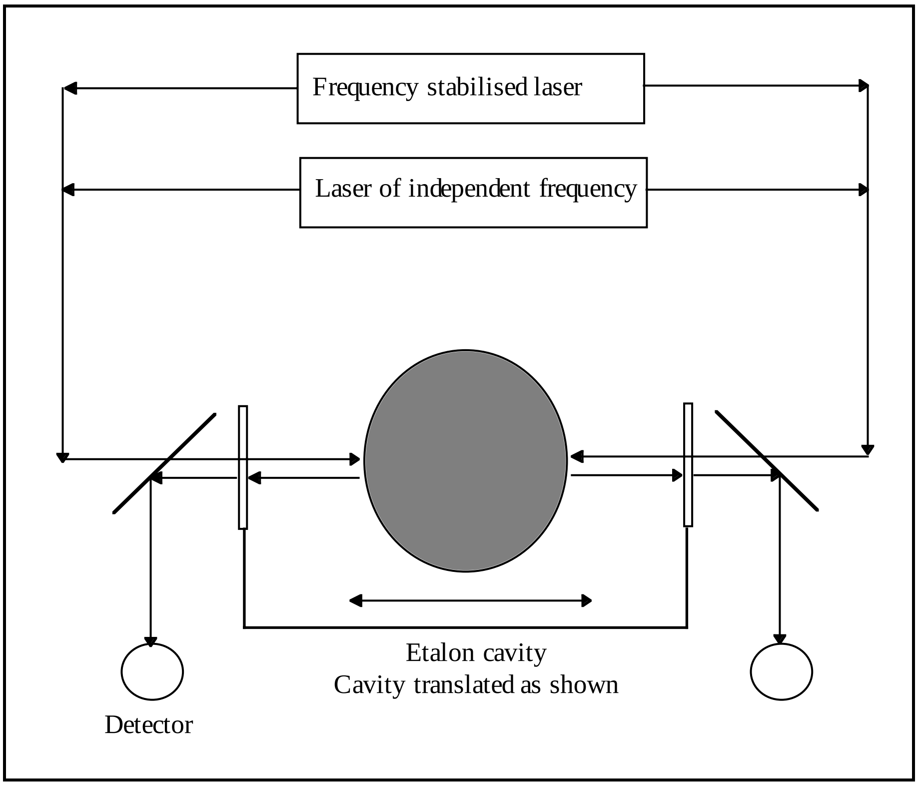


**Figure 11: Diagram illustrating technique of determining diameter of the silicon sphere.**

The apparatus is shown in more detail in figure 12. The etalon is scanned using a piezoelectric transducer (PZT), and the displacement is measured approximately using the linear variable differential transformer (LVDT). The whole apparatus was kept in a temperature controlled room, with the sphere and etalon in a water jacket. The temperature fluctuations were thus kept less than 2mK. Using this technique, the diameter of the sphere was measured with standard deviation 8.6nm, that is of 0.28 parts per million uncertainty in the volume.[21]

[21] K. Fujii, M. Tanaka, Y. Nezu, K. Nakayama, R. Maui, 1993, *Accurate Determination of the Density of a Crystal Silicon Sphere*, IEEE Trans. Instrum. Meas. **42** (2) p395

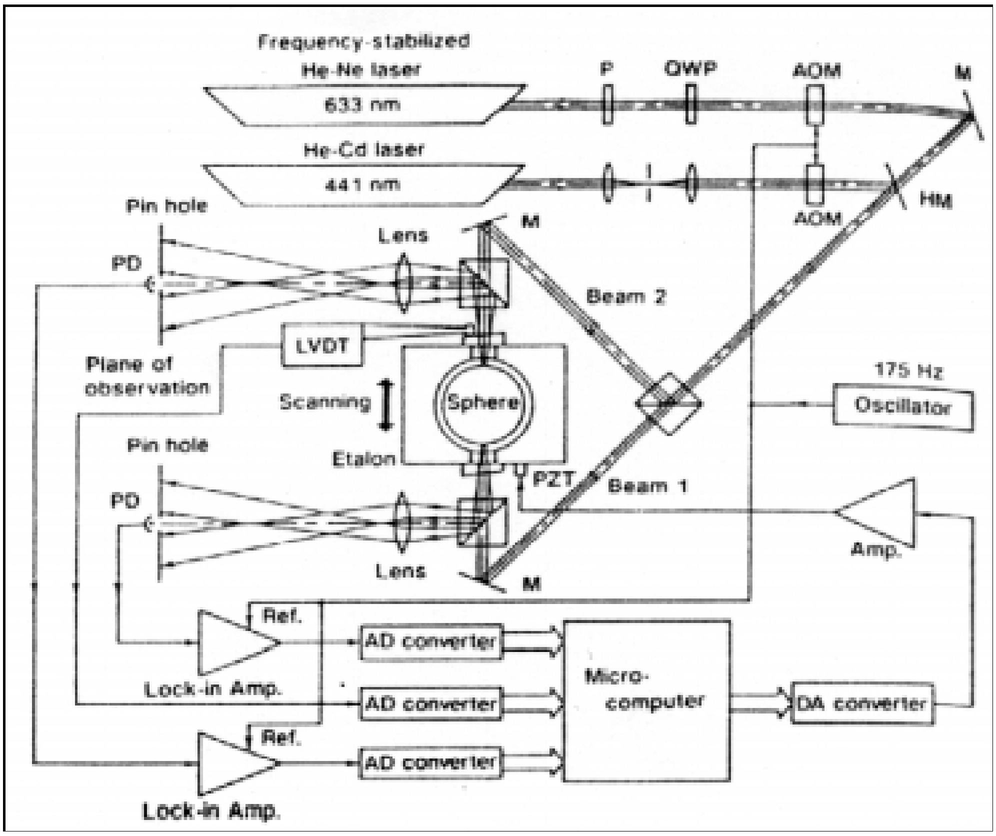


**Figure 12: Apparatus for the precision determination of the diameter of a silicon sphere.** P polarizer, QWP quarter wave plate, M mirror, HM half mirror, PD photodetector, AOM acousto-optic modulator, PZT piezoelectric transducer, LVDT linear variable differential transformer

The only measurement that remains is that of comparing the mass of the silicon sphere with that of the platinum-iridium standards used today to determine and monitor the kilogram. This is not trivial, since the density of silicon is considerably different to that of platinum (volume of platinum-iridium kilogram is 4.66 x $10^{-5}m^3$, whereas volume of silicon crystal is typically 4.30 x $10^{-4}m^3$), and the weight measurement needs to be conducted in air to ensure continuity with historical weighings of the kilogram. Two effects need to be taken into account - the buoyancy of the weights, which requires a knowledge of the density or air, and also the effects on the measurement of adsorption of air molecules onto the surface of the standards. These two considerations can be taken into account using a method known as the sinker system.[22]

In this technique, three nominal 1kg masses are made of each material, one of each of the shapes shown in figure 13.

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[22] M. Gläser, R. Schwartz, M. Mecke, 1991, *Experimental Determination of Air Density Using a 1kg Mass Comparator in Vacuum*, Metrologia **28** p45

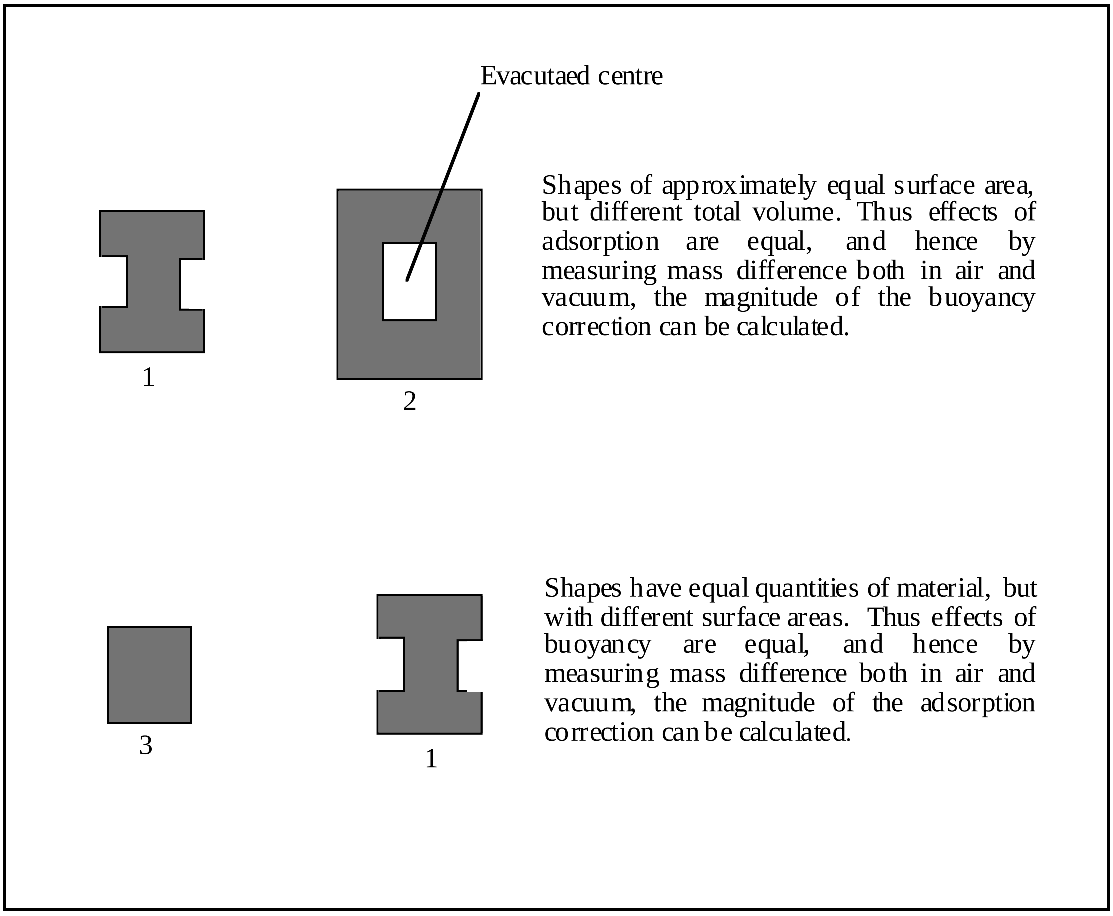


**Figure 13: Illustrating the "sinker system" used to determine the corrections needed to compare the weight objects of considerably different density.**

Even with this method, the corrections are only known to about one part in 20 000, and hence this procedure is the principal source of error in the Avogadro project to date. It is clear that for such a procedure to be used to realise the kilogram, considerable improvement is required in the accuracy of each of the measurements.

*Conclusion*

The kilogram serves as one of the centrepoints of the Système International d'Unités. Active research to replace its current definition is being conducted, and considerable progress has been made. Nevertheless, it will probably be some time before the accuracy of such measurement is sufficient to enable a re-definition of the unit of mass. In the meantime the kilogram is the only unit still to be defined in terms of an artefact standard. It is thus to be hoped that nothing untoward happens to it before a replacement can be found...